INdAM Workshop

on

Multiscale Problems: Modeling, Adaptive Discretization, Stabilization, Solvers

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Book of Abstracts

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Contents

Modeling and Numerical Analysis of Masonry Structures

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ABSTRACT

Masonry structures constitute a large component of the built environment and often form part of the national heritage. The development of reliable techniques for the numerical simulation of the behaviour of such systems is of considerable importance. We assume that masonry structures may be modelled as an aggregration of bodies in non-penetrative frictional contact. The motion of an individual body is assumed to be governed by the equations of linear elasto-dynamics leading to a large scale system of linear equations subject to inequality constraints. We compare the numerical performance of a number of alternative methods for solving the discrete system, and show how certain formulations permit a natural construction of an upper a posteriori error bound. Dynamic simulations of relevant benchmark problems will be presented.

Asymptotic Energy Analysis of Shell Eigenvalue Problems

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ABSTRACT

In the present contribution we address the study of the first vibration mode of shell structures, for small values of the relative thickness. After a brief introduction to shell problems and the difficulties related to its finite element analysis, we present the main result of the talk. Following an abstract approach and applying the Real Interpolation Theory, we show how the behavior of the smallest eigenvalue and the bending-membrane energy proportion of the respective eigenfunction can be predicted. In particular, the result shows that, even in inextensional shell problems, the bending energy of the first vibration mode is non negligible; this has large consequences on the finite element analysis of the problem. In the last part of the talk, we address the particular case of a clamped cylindrical shell. Such benchmark problem, which we are able to analyze also using Fourier expansions and scaling arguments, represents both a further investigation and a comparison for the aforementioned general result. Finally a set of numerical tests is shown.

A Posteriori Error Analisys of Finite Element Approximations of Quasi-Newtonian Flows

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ABSTRACT

We develop upper and lower a posteriori error bounds for mixed finite element approximations of a general family of steady, viscous, incompressible quasi-Newtonian flows in a bounded Lipschitz domain $\Omega \subset \mathbb{R}^d$; the family includes degenerate models such as the power-law model, as well as nondegenerate ones such as the Carreau model.

Partial differential equations with nonlinearities of the kind considered arise in numerous application areas, including geophysical models of the lithosphere, as well as chemical engineering, particularly in the modelling of the flows of pastes and dies.

The unified theoretical framework developed herein yields two-sided residual-based *a posteriori* bounds which measure the error in the approximation of the velocity in the $W^{1,r}(\Omega)$ norm and that of the pressure in the $L^{r'}(\Omega)$ norm, with $1/r+1/r' = 1$, $r \in (1, \infty)$. The analysis invokes special quasinorms, defined so as to enable dealing with singular and degenerate problems. In particular, our a posteriori error bounds include all of the residual-based *a posteriori* error bounds which have been proposed in the literature to date for special choices of the nonlinear viscosity model.

Numerical experiments indicate that for certain nonlinear viscosity models the upper and lower bounds are not robust. Nevertheless, the adapted meshes that can be obtained by applying the proposed *a posteriori* error bounds captures the relevant flow-structures and singularities in the solution. In particular, the bounds induce mesh refinement in parts of the computational domain where singular behaviour is detected, and coarsening in regions away from local singularities in the solution or the model.

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Mimetic Finite Differences Methods

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ABSTRACT

The Mimetic Finite Difference Method (MFDM) is a rather recent approach that could be classified, roughly speaking, in between Mixed Finite Element Methods and Finite Volumes.

At the moment, the main advantage of MFDM consists in the easier treatment of very complex geometrical situations, including general polyhedral elements with curved faces, which can occur, for instance, when dealing with heterogeneous materials having a geometrically complicated substructure, or during the coarsening phase of a self-adaptive decomposition process.

The talk will overview the basic features of Mimetic Finite Difference Methods, taking as a model problem Darcys law for filtration in porous media. After a rather general presentation, we will concentrate on the basic ideas that constitute the back- bone of the method, and discuss the main underlying mathematical structures. We shall also present some recent numerical tricks that make the implementation easier and cheaper.

The talk is mostly based on several papers written in collaboration with K. Lipnikov and M. Shashkov from Los Alamos N.L. and V. Simoncini from the University of Bologna.

A Multiscale Discontinuous Galerkin Method for Convection Diffusion Problems

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ABSTRACT

Proliferation of degrees-of-freedom has plagued discontinuous Galerkin methodology from its inception over 30 years ago. In this talk, we develop the numerical analysis for the new computational formulation proposed in [1] that combines the advantages of discontinuous Galerkin methods with the data structure of their continuous Galerkin counterparts.

The new method uses local, element-wise problems to develop a transformation between the parameterization of the discontinuous space and a related, smaller, continuous space. The transformation enables a direct construction of the global matrix problem in terms of the degrees-of freedom of the continuous space. This algorithm can be interpreted as a discontinuous Galerkin method where the standard piecewise polynomial spaces are replaced by the smaller interscale transfer spaces which emanate from the solutions of the local problems. We prove that this reduction does not affect the approximation, i.e., we establish optimal approximation properties for the "interscale transfer spaces".

When dealing with convection dominated problems,the issue of stability arises and we propose a stabilized version of the MDG method by adding consistent stabilizing terms of SUPG type to the global problem, without modifying the local problems, i.e., the construction of the interscale transfer spaces. The stabilized method is proved to be stable and to enjoy optimal error estimates in the stabilized norm, i.e., ensuring an optimal control on the stream-line derivative for discrete solutions.

Finally, it is known that the use of discontinuous Galerkin techniques ensures control of the streamline derivate even without a SUPG stabilization. We test numerically (by an inf-sup test) that, for simple geometries and meshes, also the MDG method provides control on the streamline derivative and the stabilization is not needed. The proof of a general stability result remains open.

Numerical tests validating our theory will be provided.

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A Minimal Stabilisation Procedure for Discontinuous Galerkin Approximations of Advection-Reaction Equations

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ABSTRACT

In this talk we will discuss stabilization mechanisms for the finite element approximation of the advection-reaction equation. On a simple model problem we review why stabilization is needed from the point of view of analysis. We then discuss how different stabilized methods introduce the stability needed for optimal a priori error estimates. In a two-dimensional framework we compare the interior penalty method using continuous approximation and discontinuous approximation. Finally we show that for higher polynomial orders the interior penalty stabilization for discontinuous approximation spaces may be relaxed. Indeed by applying a projection of the jump onto the highest 2/3 of the polynomial spectrum and applying the penalization only to the projection of the jump a method is obtained with improved mass conservation properties. The stability analysis of the modified method will be discussed and some preliminary numerical results presented.

Multiscale Problems in Computational Medicine

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ABSTRACT

At ZIB, computational medicine has been a major research area for quite a number of years. Multiscale issues arise in nearly every of the corresponding problems. The talk will present details for two selected topics:

(A) Regional hyperthermia. This is a cancer treatment where heat is to be localized in the tumor, but nowhere else in healthy tissue. The heating is produced by radiowave antennas. The wavelength in water is of the size of the body geometry – with the consequence that the full Maxwell equations must be solved; this is done by adaptive multilevel methods. As for the distribution of the heat in the body, a three-scale model is under investigation that should replace the formerly used bioheat transfer (BHT) equation approach. On the microscale, homogenization techniques play a role, which is work done together with R. Hochmuth. Recently, the traditional L_2 optimization with penalty has been replaced by the medically more interesting L_∞ optimization for the antenna parameters.

This part is joint work with M. Weiser, A. Schiela, and T. Gaenzler.

(B) Electrocardiology. This work has been started two years ago together with P. Colli Franzone, L. Pavarino, J. Lang, and B. Erdmann. The multiscale features of the time dependent monodomain or bidomain models are reflected in the behavior of the applied 3D multilevel code KARDOS, which realizes adaptivity in both time and space in the frame of an adaptive Rothe method (first time, then space discretization). I n the beginning, only unrealistic geometries, though in 3D, have been used. More recent results on defibrillation in cooperation with A. Tveito should be ready before the conference.

Scalable FETI Based Algorithms for Numerical Solution of Variational Inequalities

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ABSTRACT

We shall first briefly review the FETI based domain decomposition methodology adapted to the solution of variational inequalities such as those that describe equilibrium of a system of bodies in mutual contact. We shall consider also variational inequalities of the second type (with dissipative term) such as those describing a given (Tresca) friction. As a result, we shall obtain a convex quadratic programming problem with a special inequality and equality constraints. Using the classical results by Mandel concerning the solution of linear elliptic boundary value problems, we shall show that the condition number of the Hessian of the quadratic form is independent on the discretization parameter h.

Then we shall present two algorithms for the solution of resulting quadratic programming problems. The unique feature of these algorithms is their capability to solve the class of quadratic programming problems with spectrum in a given positive interval in O(1) iterations. The algorithms enjoy the rate of convergence that is independent of conditioning of constraints and the results are valid even for linearly dependent equality constraints.

Finally we put together our results on approximation of variational inequalities and those on quadratic programming to develop algorithms for the solution of both coercive and semi-coercive variational inequalities. We shall show that the algorithms are scalable. Rather surprisingly, the results are qualitatively the same as the classical results on scalability of FETI for linear elliptic problems. We give results of numerical experiments with parallel solution of both coercive and semicoercive problems discretized by up to more than eight million of nodal variables to demonstrate numerically scalability of the algorithms presented that was predicted by the theory.

Multiscale Finite Element Methods for Flows in Heterogeneous Porous Media and Applications

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ABSTRACT

In this talk, I will describe multiscale finite element methods for flows in heterogeneous porous media. The main idea of these multiscale finite element methods is to construct finite element basis functions that can capture the small scale information. The method has similarities with other subgrid capturing methods and subgrid stabilization techniques. I will be interested in heterogeneities that often arise in porous media applications (related to petroleum reservoirs) where one can not assume scale separation. In this case, I will present an extension of multiscale finite element method that uses some type of limited global information to take into account the global connectivity of the media. The latter removes resonance errors observed in multiscale methods that only employ local information. I will also briefly mention the extension of the method to nonlinear partial differential equation (pseudo-monotone operators), the convergence results as well as homogenization results.

This is a joint work with T. Hou and V. Ginting.

Heterogeneous Multiscale Methods

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ABSTRACT

Continuum simulations of solids or fluids for which some atomistic information is needed are typical example of multi-scale problems with very large ranges of scales. For such problems it is necessary to restrict the simulations on the micro-scale to smaller samples of the full computational domain. The heterogeneous multi-scale method is a framework for developing and analyzing numerical methods that couple computations from very different scales. Local micro-scale simulations on small domains supply missing data to a macro-scale simulation on the full domain. Examples are local molecular dynamics or kinetic Monte Carlo computations that produce data to a continuum macro-scale model. Application to epitaxial growth will be discussed.

Simulation with Adaptive Modeling

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ABSTRACT

In many fields of scientific computing, the underlying equations are well- known but involve models of different complexity, scales, and accuracy. In various cases, the most accurate and validated model cannot be chosen in numerical simulations because of the large amount of computational costs. Simpler models usually need less computing time and involve less couplings between variables. Therefore, it is desirable to apply detailed models only in those regions of the computational domain where necessary. It is also important to balance modeling errors with discretization errors to avoid situations in which, for instance, a crude model is employed together with an over-refined mesh.

In the first part of the talk, we present a general methodology based on dual-weighted a posteriori error estimates that yield, for a given finite element solution and a given output functional, model error indicators as well as discretization error indicators [1]. Using these indicators, an adaptive strategy coupling multimodeling with local mesh refinement is designed to equilibrate both sources of error. The performances of the methodology are assessed on various academic test cases.

In the second part of the talk, we apply the twofold adaptive method to the numerical computation of laminar flames with finite-rate chemistry [2]. The part of the model that is adaptively modified is that describing multicomponent diffusion processes. The simple model employs a Fick-type law to evaluate the diffusion flux of each chemical species, while the detailed models also accounts for cross-diffusion processes and for the Soret effect (mass diffusion driven by temperature gradients). Examples of flame structures which are sensitive to the Soret effect are presented.

Finally, we present two nonstandard extensions of the analysis. First, we design and analyze a setting where the model dimension can be varied [3]; targeted applications include flows in river or estuaries and blood circulation models. Second, we consider polymeric fluid flows where the simple model is purely macroscopic (e.g., the Oldroyd-B model), whereas the detailed model involves micro-macro couplings in which the evolution of polymer chains is governed by a stochastic PDE [4].

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The Finite Element Immersed Boundary Method: Model, Stability, and Numerical Results

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ABSTRACT

The Immersed Boundary Method (IBM) has been designed by Peskin for the modeling and the numerical approximation of fluid-structure interaction problems (see [1] for a nice review). Recently, a finite element version of the IBM has been developed (see [2], [3], [4], [5]) which offers interesting features for both the analysis of the problem under consideration and the robustness and flexibility of the numerical scheme.

In this talk we shall review our model and present numerical results in two and three dimensions fully confirming the good performance of our scheme.

A preliminary stability analysis shows a condition linking the time step size and the discretization parameter along the immersed boundary. This condition is confirmed by our numerical experiments.

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Extending the Scalability of FETI-DP: from Exact to Inexact Algorithms

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ABSTRACT

Dual-primal FETI methods are among the most severely tested domain decomposition methods for the solution of partial differential equations. With the advent of massively parallel computers with up to 100,000 processors new concepts are necessary to maintain the good scalability of these iterative methods. In this talk, a framework for the algorithmic design of dual-primal FETI methods will be discussed focusing on the construction of scalable domain decomposition methods on massively parallel machines; preliminary numerical results will also be presented.

This work is based on joint work with Oliver Rheinbach, Essen.

Stabilization Mechanisms in Discontinuous Galerkin Methods. Applications to Advection-Diffusion-Reaction Problems

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ABSTRACT

In recent years Discontinuous Galerkin methods (DG), originally introduced by Reed and Hill for pure hyperbolic problems, have become increasingly popular in many other fields of application. One of the reasons of their success is probably due to their versatility to deal with complicated geometries without requiring special conditions on the mesh. Another important feature is that they are locally conservative, a nonnegligible property. The counterpart is their lack of stability, which is in general imposed through a penalty on the jumps at the interfaces.

In this talk we discuss a new perspective on the construction of DG methods for linear partial differential equations. They are interpreted as methods that define an approximation by means of a variational formulation enforcing the partial differential equation together with the boundary and continuity conditions satisfied by the exact solution. In this way, the methods establish a linear relationship between the residual of the approximation inside each element and the jumps across interelement boundaries. Thus, we devise a simple and general approach to ensure the desired stability properties of the resulting methods. In doing so, we identify the ingredients which enforce those properties and ensure stability. The conclusion is that, in the context of DG methods, the use of jump penalties, upwinding, and Hughes- Franca type residual-based stabilizations are all different forms of the same mechanism. (We refer to [1] for a detailed discussion).

Finally, we apply this framework to design DG-approximations of advection diffusion-reaction problems.

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High Order Relaxed Schemes for non Linear Diffusion Problems

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ABSTRACT

Recently several relaxation approximations to partial differential equations have been proposed. Examples include conservation laws, Hamilton-Jacobi equations, convection-diffusion problems, gas dynamics problems (see e.g. [1, 2, 3, 4, 5, 6, 7]). The present work focuses onto diffusive relaxed schemes for the numerical approximation of nonlinear parabolic equations. These schemes are based on suitable semilinear hyperbolic system with relaxation terms. High order methods are obtained by coupling ENO and WENO schemes for space discretization with IMEX schemes for time integration. Error estimates and convergence analysis are developed for semidiscrete schemes with numerical analysis for fully discrete relaxed schemes. Some numerical examples in two dimension illustrate the high accuracy and good properties of the proposed numerical schemes. Moreover, the proposed schemes can be easily implemented for parallel computer and applied to more general system of nonlinear parabolic equations in two- and three-dimensional cases. We also show the numerical simulation of a more complex problem araising in 3D vasculogenesis by using a suitable hyperbolic-parabolic system.

This talk is based on joint works [8, 9] with F. Cavalli, A. Gamba, G. Puppo, M. Semplice, and G. Serini.

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Multiphysics in Haemodynamics: Fluid-structure Interaction between Blood and Arterial Wall

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ABSTRACT

The mechanical interaction between blood and arterial wall is the key mechanism, responsible for the propagation of pressure waves from the heart to peripheral vessels and has a fundamental role in regulating blood pressure in the whole cardiovascular system.

Such interaction introduces different spacial and temporal scales, since the pressure propagation speed and wavelength are much larger than the characteristic blood velocity and size of large arteries.

This multiscale nature induces some difficulties at the numerical level. In this talk we will mainly focus on numerical algorithms for simulating fluid-structure interaction in a single arterial segment. We will show why explicit or loosely coupled FSI time advancing schemes, which solve sequentially the fluid and the structure subproblems at each time step, are unstable when applied to this problem. We will review, then, some stable implicit and semi-implicit coupled algorithms that are well suited in this context.

We will also address the problem of finding proper boundary conditions capable of absorbing the outgoing traveling pressure waves and suggest the use of non linear resistive-type conditions.

Convergence and Optimal Complexity of AFEM for General Elliptic Operators

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ABSTRACT

The chief difficulty to prove convergence of adaptive finite element methods (AFEM) for general (non-symmetric) second order linear elliptic PDE is that the energy error and oscillation do not decouple. We regard the sum of energy error and oscillation, the so-called total error, as the key quantity for AFEM to control. We thus use a Dorfler marking for the sum of local error indicators and oscillation, followed by a new Dorfler marking for oscillation relative to the sum of error estimator plus oscillation. We prove that the resulting AFEM is a contraction for the total error between two consecutive adaptive loops. This strict error reduction property then leads to optimal complexity of AFEM, that is to a total error decay in terms of number of degrees of freedom as predicted by the best approximation.

This work is joint with J.M. Cascon.

Hybrid Multiscale Methods for Hyperbolic and Kinetic Equations

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ABSTRACT

We consider the development of hybrid numerical methods for the solution of hyperbolic and kinetic problems with multiple scales. The main motivation is the need to couple macroscopic and microscopic model in all cases where the macroscopic model is not sufficiently accurate. The key aspect in the development of the algorithms is the choice of a suitable hybrid representation of the solution and a merging of Monte Carlo methods in non-equilibrium regions, where we need the solution of the microscopic model, with deterministic methods in equilibrium regions, where a macroscopic model provides a correct description. Applications to hyperbolic relaxation systems and to Boltzmann-BGK equations are presented to show the performance of the new methods.

Discontinuous Galerkin Approximation of the Maxwell Eigenproblem

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ABSTRACT

One of the most relevant problems in computational electromagnetics is the computation of the eigenfrequencies of the Maxwell equations in a cavity. Finite element techniques are widely used to approximate this problem, and, in recent years, a complete mathematical theory has been developed for conforming approximations, identifying the properties that the underlying finite element spaces need to fulfill in order to guarantee spurious-free approximations. On the other hand, the use of discontinuous Galerkin (DG) methods in electromagnetism is attracting thanks to their flexibility in the mesh design and in the choice of shape functions. The main difficulties encountered in the analysis of DG approximations of the Maxwell equations are related to the lack of ellipticity and underlying compactness property of the Maxwell operator, which is "amplified" by the use of non-conforming approximation spaces.

In this talk, a theoretical framework for the analysis of DG approximations of the Maxwell eigenproblem with discontinuous coefficients will be presented. Necessary and sufficient conditions for a spurious-free approximation are established, and it is shown that basically all the discontinuous Galerkin methods in the literature, at least on conforming meshes, fit into this framework; the actual possibility of obtaining spurious-free DG approximations on meshes with hanging nodes will be discussed. The relations with the classical theory for conforming approximations will be pointed out. Well-posedness and quasi-optimal error estimates for DG discretizations (for sufficiently fine meshes) of the Maxwell source problem are derived as a direct consequence of the developed spectral theory.

The results presented in this talk have been obtained in collaboration with Annalisa Buffa, IMATI-CNR, Pavia.

The Fine-scale Green's Function

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ABSTRACT

We derive an explicit formula for the fine-scale Green's function arising in variational multiscale analysis. The formula is expressed in terms of the classical Green's function and a projector which defines the decomposition of the solution into coarse and fine scales. The theory is presented in an abstract operator format and subsequently specialized for the advection-diffusion equation. It is shown that different projectors lead to fine-scale Green's functions with very different properties. For example, in the advection-dominated case, the projector induced by the H^1_0-seminorm produces a fine-scale Green's function which is highly attenuated and localized. These are very desirable properties in a multiscale method, and ones that are not shared by the L^2-projector. By design, the coarse-scale solution attains optimality in the norm associated with the projector. This property, combined with a localized fine-scale Green's function, indicates the possibility of effective methods with local character for dominantly hyperbolic problems. The constructs lead to a new class of stabilized methods, and the relationship between H^1_0-optimality and SUPG is described.

Multi Phase Field Simulations of Solid-solid Phase Transitions in Steel

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ABSTRACT

Solid-solid phase transitions in steel are macroscopically observable phenomena which are based on micro- or mesoscopic changes. On the scale of grains in the material, phases are clearly separated, while on the macro scale, the material behaviour is often described by averaged phase fractions. We will present numerical models and simulations, where the phase transitions in different grains are described by different phase field functions. The implementation is based on the adaptive finite element toolbox ALBERTA.

This is joint work with T. Moshagen and M. Bhm, and partially supported by DFG via SFB 570 "Distortion engineering".

A Basic Convergence Result for Adaptive Finite Elements

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ABSTRACT

Adaptive finite element methods are successfully used for the numerical solution of partial differential equations since the 1970th. Nowadays they are standard tools in science and engineering. Only adaptive methods allow a simulation of multi-scale problems, especially in three space dimensions. The typical adaptive iteration is a loop of the form

$$\text{Solve} \quad \rightarrow \quad \text{Estimate} \quad \rightarrow \quad \text{Mark} \quad \rightarrow \quad \text{Refine.}$$

The analysis of adaptive method has to provide computable error bounds for the step 'Estimate', the so-called a posteriori error estimators. Secondly, it has to be analyzed, if such an adaptive iteration really leads to improved discrete solutions, i. e. it has to be proven that the sequence of discrete solutions converges to the exact solution.

Up to now, convergence results for adaptive finite element methods in higher space dimensions rely on special marking procedures and special refinement rules that are not used in practical applications, e. g. by engineers. In this talk we will show that all practical adaptive algorithms converge for a large class of problems without relying on any special ingredients or a guarenteed error reduction.

This is joint work with Pedro Morin (Universidad Nacional del Litoral, Argentina) and Andreas Veeser (Università degli Studi di Milano, Italy).

Reduced Models for Haemodynamics Problems and Their Coupling

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ABSTRACT

In this talk we address the issue of selecting suitable models for blood flow in the arterial system which are accurate enough for the problem at hand and computationally effective. We will present the derivation of one dimensional models for the main arterial tree based on the solution of hyperbolic systems of equation. We will give some results on the stability and existence of regular solutions for the model of a singe artery and discuss the issues related to the coupling of the basic model to form a network representing the main arterial tree. Some results about the coupling of the reduced model with a local model based on the solution of the 3D fluid-structure interaction problem will be presented as well.

Extending the Theory for Iterative Substructuring Algorithms to less Regular Subdomains

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ABSTRACT

Assumptions on the regularity of the subdomains are routinely made in developing bounds for the condition numbers of a variety of iterative substructuring methods such as those in the FETI and BDDC families. A number of technical tools have thus been developed for subregions which are unions of a few shape-regular large tetrahedra. In this talk, some of these tools will be extended to subregions such as those which arise when using mesh partitioners. Examples will also be given of algorithms for which the new tools will provide new insight.

Propagation, Dispersion, Control, and Numerical Approximation of Waves

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ABSTRACT

In this lecture we shall discuss several topics related with numerical approximation of waves.

Control Theory is by now and old subject, ubiquitous in many areas of Science and Technology. There is a quite well-established finite-dimensional theory and many progresses have been done also in the context of PDE (Partial Differential Equations). But gluing these two pieces together is often a hard task from a mathematical point of view.

This is not a merely mathematical problem since it affects modelling and computational issues. In particular, the following two questions arise: Are finite-dimensional and infinite-dimensional models equally efficient from a control theoretical point of view? Are controls built for finite-dimensional numerical schemes efficient at the continuous level?

In this talk we shall briefly analyze these issues for the wave equation as a model example of propagation without damping. We shall show that high frequency spurious oscillations may produce the divergence of the most natural numerical schemes. This confirms the fact that finite and infinite-dimensional modelling may give completely different results from the point of view of control. We shall then discuss some remedies like filtering of high frequencies, multi-grid techniques and numerical viscosity.

Similar questions arise when building numerical approximation schemes for nonlinear Schrödinger equations.

We first consider finite-difference space semi-discretizations and show that the standard conservative scheme does not reproduce at the discrete level the properties of the continuous wave and Schrödinger equations. This is due to high frequency numerical spurious solutions. In order to damp out these high-frequencies and to reflect the properties of the continuous problem we introduce a two-grid preconditioner. We prove that the propagation and dispersion properties of this two-grid scheme are uniform when the mesh-size tends to zero. Finally we prove the convergence of this numerical scheme both for the control of teh wave equation and for the Cauchy problem for a class of nonlinear Schrödinger equations with nonlinearities that may not be handeled by standard energy methods and that require the so-called Strichartz inequalities.