EXERCISE 1

- Write Matlab functions of the form `[x,res,its] = Jacobi(A,b,tol,maxit,x0)` that implement the Jacobi method. Do the same for the and Gauss-Seidel method. Here `res` is the vector of residual norms for each iteration. Take the following algorithm as reference (as stopping criteria, use $\|r_k\| / \|r_0\| <$`tol` and `its` $>$ `maxits`).

  Chosen M and N (as in $A = M - N$) and x_0, compute r_0.
  For $k = 1, 2, \dots,$ until convergence:
  $x_k = x_{k-1} + M^{-1} r_{k-1}$ (do not compute the matrix M^{-1}, just solve the system)
  $r_k = b - Ax_k$

  Useful Matlab commands: `M = diag(diag(A))`, `M = tril(A)`.

- Test the two methods on the problem $Ax = b$, where

$$A = \begin{bmatrix} 7 & 6 & 3 \\ 2 & 5 & -4 \\ -4 & -3 & 8 \end{bmatrix}, \qquad b = \begin{bmatrix} 16 \\ 3 \\ 1 \end{bmatrix},$$

  using `tol`$= 10^{-6}$, `maxits`$= 1000$, `x0` $= (0, \dots, 0)^T$. Compare the history of convergence of the two methods, i.e. plot the iteration number vs. the norm of the residual. What is the value of $\max_i |\lambda_i(B)|$, where B is the iteration matrix?

EXERCISE 2

Consider the system

$$Ax = b$$

with $A = \texttt{gallery('poisson',n)}$ and $b = (1, \ldots, 1)^T$.

- For $n = 40$ solve the system using the Jacobi, Gauss-Seidel and CG (as implemented by the `pcg` function) methods. Compare the history of convergence of the three methods.

- For $n = 10, 20, \ldots, 100$ solve the system using CG. Check how the number of iterations varies, by plotting the number of iterations vs. n. Check also how the condition number $\lambda_{\max}(A)/\lambda_{\min}(A)$ varies (use the function `eigs`), and try to relate it with the number of iterations.