Numerical Methods in Engineering Sciences
14/7/2021

Written Exam

First name: _____
Last name: _____

Student ID: _____

☐ I want to take the BASIC EXAM
☐ I want to take the ADVANCED EXAM

Exam rules:

- Basic exam: the maximum grade is 24/30.

- Advanced exam: the maximum grade is 30/30 cum laude.

Total time is 1 hour. Students who get a positive grade in the written part (i.e., at least 18/30) *might* choose to take an oral exam. For students who choose the basic written exam, the maximum grade obtainable can never exceed 24/30.

## BASIC EXAM

**1.** Write the pseudocode of the backward substitution method used to solve linear systems where the matrix is upper triangular. Show how it works when solving the system $Ax = b$, where

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}, \qquad b = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}.$$

**2.** Describe the implicit Euler method. Then, given the Cauchy problem:

$$\begin{cases} y'(t) = 3y(t) + 2t^2 \text{ for } t > 0 \\ y(0) = 1; \end{cases}$$

compute two steps by the implicit Euler method, with $\Delta t = 2$, in order to approximate $y(4)$. Report the intermediate computations.

## ADVANCED EXAM

**3.** Write the pseudocode of the backward substitution method used to solve linear systems where the matrix is upper triangular. Describe (with full justification) its computational cost. Show how it works when solving the system $Ax = b$, where

$$A = \begin{bmatrix} 2 & 2 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 4 \end{bmatrix}, \qquad b = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}.$$

**4.** Describe the descent methods: the Gradient (steepest descent) and Conjugate Gradient methods