Numerical Methods in Engineering Sciences
25/2/2022

Written Exam

First name: _____
Last name: _____

Student ID: _____

☐ I want to take the BASIC EXAM
☐ I want to take the ADVANCED EXAM

Exam rules:

- Basic exam: the maximum grade is 24/30.
- Advanced exam: the maximum grade is 30/30 cum laude.

Total time is 1 hour. Students who get a positive grade in the written part (i.e., at least 18/30) *might* choose to take an oral exam. For students who choose the basic written exam, the maximum grade obtainable can never exceed 24/30.

BASIC EXAM

1. Given the function $f(x) = x^2 + x^3$ compute its Lagrange interpolant of degree 2 through the points $x_1 = -2$, $x_2 = 0$, $x_3 = 3$.

2. Write the pseudocode of the backward substitution method used to solve linear systems where the matrix is upper triangular. Describe (with full justification) its computational cost. Show how it works when solving the system $Ax = b$, where

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 4 \end{bmatrix}, \qquad b = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}.$$

ADVANCED EXAM

3. Write the pseudocode of the Gaussian elimination method, without pivoting, and apply it to solve the linear system

$$\begin{bmatrix} 3 & -2 & 1 \\ 2 & 2 & 2 \\ 2 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 21 \end{bmatrix}$$

showing the intermediate computations.

4. Give the statement of the convergence theorem for the Newton method used to solve nonlinear equations. Prove that the order of convergence of the methods is 2.