

- I want to take the BASIC EXAM  
 I want to take the ADVANCED EXAM

Exam rules:

- Basic exam: the maximum grade is 24/30.
- Advanced exam: the maximum grade is 30/30 cum laude.

Total time is 1 hour. Students who get a positive grade in the written part (i.e., at least 18/30) *might* choose to take an oral exam. For students who choose the basic written exam, the maximum grade obtainable can never exceed 24/30.

**BASIC EXAM**

1. Given the Cauchy problem:

$$\begin{cases} y'(t) = -2y(t) + t^2 \text{ for } t > 0 \\ y(0) = 2; \end{cases}$$

compute two steps by the implicit Euler method, with  $\Delta t = 1/2$ , in order to approximate  $y(1)$ . Report the intermediate computations.

2. Introduce the power method for the computation of the dominant eigenvalue and eigenvector, with pseudocode. State conditions that guarantee its convergence

**ADVANCED EXAM**

3. Describe the Crank-Nicolson scheme for the solution of an ODE and explain its relation with the trapezoidal quadrature rule. Then, compute one step of the Crank-Nicolson scheme for the problem

$$\begin{cases} y'(t) = (t + 1) y(t) \\ y(0) = 1 \end{cases}$$

selecting  $\Delta t = 2$ .

4. State and prove the theorem on the existence and uniqueness of the Lagrange interpolant of a given function.