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Geometria Algebrica. - *A remark on the Picard group of spin moduli space.* Nota (*) del Corrisp. Maurizio Cornalba.

Abstract. - We describe a number of classes in the Picard group of spin moduli space and determine the relations they satisfy; as an application, we show that the Picard group in question contains 4-torsion elements.

Key Words: Moduli; Algebraic curves; Theta-characteristics.

Riassunto. - *Una osservazione sul gruppo di Picard dello spazio dei moduli delle curve con struttura di spin.* Si descrivono varie classi nel gruppo di Picard dello spazio dei moduli delle curve con struttura di spin e si determinano le relazioni che esse soddisfano; come applicazione, si mostra che il gruppo di Picard in questione contiene elementi di ordine 4.

1. Introduction

Spin moduli space (in genus g) is the space parametrizing all couples

(smooth genus g algebraic curve C, theta-characteristic on C);

it has a natural structure of algebraic variety and will be denoted S_g. A well-behaved compactification of S_g was introduced in [1]. In the same paper several natural classes in the Picard group of this compactification (or rather of the corresponding moduli stack) were described. It is of course of interest to determine whether these classes generate the Picard group in question and what relations they satisfy. The answer to the first problem is still unknown. The second problem is easier, and we can give a complete answer to it. In fact, all relations are already described in [1], but the most remarkable among them is stated there without proof. The main purpose of this note is to provide the missing proof; as a byproduct, we describe new natural classes and determine their relation to the ones defined in [1]. We conclude by showing that the Picard group of spin moduli space contains 4-torsion. We work over the complex numbers throughout.

2. Spin curves

In this section we collect, without proof, those facts about spin moduli space and its compactification that will be relevant for our purposes, referring to [1] for details.

A *spin curve* of genus g is the datum of a semistable genus g curve X, plus an invertible sheaf ζ_X of degree $g-1$ on X and a homomorphism of invertible sheaves $\alpha_X : {\zeta_X}^{\otimes 2} \to \omega_X$ such that:

i) If we call *exceptional* those smooth rational components of X that contain only two singular points of X, then no two distinct exceptional components of X meet.
ii) The restriction of ζ_X to any exceptional component of X has degree 1.
iii) α_X is not zero at a general point of every non-exceptional component of X.

Notice that the definition forces α_X to vanish identically on all exceptional components of X and to be an isomorphism elsewhere. The datum of a ζ_X and an α_X satisfying *ii)* and *iii)* is called a *spin structure* on X. Clearly, a spin curve such that X is smooth amounts to the datum of X plus a choice of theta-characteristic on it. A *family of spin curves* consists of a flat family of semistable curves $f : \mathcal{X} \to B$, plus an invertible sheaf ζ_f on $\mathcal{X}$ and a homomorphism $\alpha_f : {\zeta_f}^{\otimes 2} \to \omega_f$ such that the restriction of these data to any fiber of f gives rise to a spin curve; here, and in the sequel, we write ω_f for the relative dualizing sheaf $\omega_{\mathcal{X}/B}$. Given a family of spin curves as above, one sets $\mathcal{E}_f = \omega_f \otimes \zeta_f^{-2}$. Very roughly speaking, $\mathcal{E}_f$ 'is' $\mathcal{O}(E)$, where E stands for the divisor swept by the exceptional components in the fibers of f (when this is a divisor). Let $k : \mathcal{Y} \to B$ be another family of spin curves: an isomorphism between $f : \mathcal{X} \to B$ and $k : \mathcal{Y} \to B$ consists of isomorphisms

$$h : \mathcal{X} \to \mathcal{Y} \qquad\qquad \gamma : h^*(\zeta_k) \to \zeta_f$$

such that $f = k \circ h$ and γ is compatible with the natural isomorphism between $h^*(\omega_k)$ and ω_f. Notice that this differs slightly from the notion of isomorphism used in [1]; at the end of this section and at the beginning of the next we shall explain how this affects the results of [1].

The set of isomorphism classes of spin curves of genus g is denoted $\overline{S}_g$; it carries a natural structure of algebraic variety which makes it a coarse moduli space for spin curves, and it can be shown to be projective. One defines the *parity* of a spin curve X to be the parity of $h^0(X, \zeta_X)$; in the smooth case, this reduces to the notion of parity of a theta-characteristic. As in the smooth case, parity is a deformation invariant, so $\overline{S}_g$ is the disjoint union of two connected components $\overline{S}_g^{\,ev}$ and $\overline{S}_g^{\,odd}$ consisting, respectively, of even and odd spin curves.

The boundary $\partial S_g = \overline{S}_g - S_g$ of spin moduli space is a divisor made up of irreducible components $A_i^{ev}, A_i^{odd}, B_i^{ev}, i = 0, \dots, [\frac{g}{2}]$ and $B_i^{odd}, i = 0, \dots, [\frac{g-1}{2}]$. The general members of these components are as follows:

- For A_i^{ev}(resp., B_i^{ev}), $i > 0$: two smooth components C_1 and C_2 of genera i and $g - i$, joined at points $p \in C_1$ and $q \in C_2$ by a $\mathbf{P}^1$, with even (resp., odd) theta-characteristics on C_1 and C_2 'glued' to $\mathcal{O}(1)$ on $\mathbf{P}^1$.
- For A_i^{odd} (resp., B_i^{odd}), $i > 0$: as above, but with an even (resp., odd) theta-characteristic on C_1 and an odd (resp., even) one on C_2.
- For A_0^{ev} (resp., A_0^{odd}): an irreducible curve of genus g with one node, with an even (resp., odd) spin structure.
- For B_0^{ev} (resp., B_0^{odd}): an irreducible curve of genus g with one node, blown up at the node, with an even (resp., odd) spin structure.

An isomorphism between families of spin curves $f : \mathcal{X} \to B$ and $k : \mathcal{Y} \to B$ was defined in [1] to be an isomorphism of fibre spaces $h : \mathcal{X} \to \mathcal{Y}$ such that there exists an isomorphism $\gamma : h^*(\zeta_k) \to \zeta_f$ compatible with $h^*(\omega_k) \cong \omega_f$. This differs from the convention adopted in the present paper in that the datum of γ is not included in the definition; notice however that, as we observed in [1], given h, γ is determined up to sign on each connected component of B. This change in definitions causes no essential modifications in the mathematics of [1], although, of course, the wording of some results has to be modified. Naturally, the most apparent change is in the structure of the automorphism group of a spin curve X. The 'new' automorphism group is a central extension of the 'old' one by a cyclic group of order two, this being generated by the automorphism 'multiplication by -1 in ζ_X', which will be denoted ε_X from now on. Accordingly, all the results in [1] concerning automorphisms of a spin curve X are valid in our context, provided one interprets $\mathrm{Aut}_0(X)$ to mean the group of those inessential automorphisms of the semistable curve underlying X which come from automorphisms of X (an automorphism is said to be *inessential* if it restricts to the identity on the complement of the exceptional components).

3. Natural divisor classes

We denote by $\overline{\mathcal{S}}_g$, $\overline{\mathcal{S}}_g^{ev}$, $\mathcal{S}_g$, etc., the moduli stacks of genus g spin curves, even genus g spin curves, smooth genus g spin curves, and so on. A *line bundle on* $\overline{\mathcal{S}}_g$ is the datum of a line bundle L_f on B for every family $f : \mathcal{X} \to B$ of genus g spin curves and of an isomorphism between $h^*(L_k)$ and L_f for every cartesian diagram of families of spin curves

$$\begin{array}{ccc} \mathcal{X} & \longrightarrow & \mathcal{Y} \\ f\downarrow & & k\downarrow \\ B & \xrightarrow{h} & T \end{array}$$

These isomorphisms are required to satisfy a suitable cocycle condition (see [4], [5], or [6] for details). The Picard group $\mathrm{Pic}(\overline{\mathcal{S}}_g)$ consists of all isomorphism classes of line bundles on $\overline{\mathcal{S}}_g$. One similarly defines the notions of line bundle and Picard group for $\overline{\mathcal{S}}_g^{ev}$, $\mathcal{S}_g$, and so on. Clearly, $\mathrm{Pic}(\overline{\mathcal{S}}_g)$ is the direct sum of $\mathrm{Pic}(\overline{\mathcal{S}}_g^{ev})$ and $\mathrm{Pic}(\overline{\mathcal{S}}_g^{odd})$; if η is a class in $\mathrm{Pic}(\overline{\mathcal{S}}_g)$, we shall denote its $\mathrm{Pic}(\overline{\mathcal{S}}_g^{ev})$- and $\mathrm{Pic}(\overline{\mathcal{S}}_g^{odd})$-components by η^{ev} and η^{odd}.

The discrepancy between the definition of isomorphism of spin curves adopted here and the one adopted in [1] obviously affects Picard groups. More precisely, if L is a line bundle on $\overline{\mathcal{S}}_g$ and X is a spin curve, the automorphism ε_X acts on L_X: the 'old' Picard group is precisely the subgroup of $\mathrm{Pic}(\overline{\mathcal{S}}_g)$ consisting of the classes of all the ε-invariant line bundles L, that is, those such that the action of ε_X on L_X is trivial for any X. For us, this makes little difference, for all the divisor classes we work with are ε-invariant.

The most obvious classes in $\mathrm{Pic}(\overline{\mathcal{S}}_g)$ are the classes λ and δ_i, $i = 0, \ldots, [g/2]$, coming by pullback from the classes with the same name in $\mathrm{Pic}(\overline{\mathcal{M}}_g)$ (cf. [4] or [5]). For the line bundle L with class λ, one has that $L_f = \det f_*\omega_f = \det Rf_*\omega_f$ for any family of spin curves $f : \mathcal{X} \to B$, while the δ_i are associated to the boundary components of M_g. In [1] we mimicked these constructions, defining classes α_i^{ev}, α_i^{odd}, β_i^{ev}, and β_i^{odd}, associated to the boundary components A_i^{ev}, A_i^{odd}, B_i^{ev}, and B_i^{odd} of spin moduli space, and a class μ corresponding to the line bundle M such that $M_f = \det Rf_*\zeta_f$ for any family of spin curves $f : \mathcal{X} \to B$. To simplify notation, we set $\beta_{g/2}^{odd} = 0$ when g is even. In [1] it is shown that

$$\delta_0 = \alpha_0 + 2\beta_0 \qquad \delta_i = 2(\alpha_i + \beta_i), \quad i > 0. \tag{3.1}$$

There is another construction which yields classes in the Picard group of $\overline{\mathcal{S}}_g$. Let $f : \mathcal{X} \to B$ be a family of noded curves, and let L, M be line bundles on $\mathcal{X}$. In [2] (cf. also [3]), Deligne shows that the line bundle $\langle L, M\rangle$ on B defined by

$$\langle L, M\rangle = \det Rf_*(LM) \otimes (\det Rf_*L)^{-1} \otimes (\det Rf_*M)^{-1} \otimes \det Rf_*\mathcal{O}_{\mathcal{X}} \tag{3.2}$$

depends bilinearly on its two arguments, in the sense that there are natural isomorphisms

$$\langle L_1L_2, M\rangle \cong \langle L_1, M\rangle \otimes \langle L_2, M\rangle,$$
$$\langle L, M_1M_2\rangle \cong \langle L, M_1\rangle \otimes \langle L, M_2\rangle,$$

satisfying suitable compatibility conditions.

When B is a smooth curve, denoting by F a general fiber of f, by b the genus of B, and by $(\ \cdot\ )$ the intersection pairing on $\mathcal{X}$, the degree of $\langle L, M\rangle$ is:

$$\begin{aligned}\deg\langle L, M\rangle &= \deg Rf_*(LM) - \deg Rf_*L - \deg Rf_*M + \deg Rf_*\mathcal{O}_{\mathcal{X}}\\ &= \chi(LM) - \chi(LM|_F)\chi(\mathcal{O}_B) - \chi(L) + \chi(L|_F)\chi(\mathcal{O}_B)\\ &\quad - \chi(M) + \chi(M|_F)\chi(\mathcal{O}_B) + \chi(\mathcal{O}_{\mathcal{X}}) - \chi(\mathcal{O}_F)\chi(\mathcal{O}_B)\\ &= \chi(LM) - \chi(L) - \chi(M) + \chi(\mathcal{O}_{\mathcal{X}})\\ &= (L\cdot M).\end{aligned}$$

Using (3.2), linearity, and the fact that, for any L, $\det Rf_*(L^{-1}\omega_f) \cong \det Rf_*L$, we find that

$$\begin{aligned}\langle L, \omega_f\rangle \otimes \langle L, L\rangle^{-1} &\cong \langle L, L^{-1}\omega_f\rangle\\ &\cong (\det Rf_*\omega_f)^2 \otimes (\det Rf_*L)^{-2}.\end{aligned}$$

This proves

$$(\det Rf_*L)^2 \cong \langle L, L\rangle \otimes \langle L, \omega_f\rangle^{-1} \otimes (\det Rf_*\omega_f)^2, \tag{3.3}$$

which can be viewed as a concrete version of the Grothendieck Riemann-Roch theorem for f and L. On the other hand, multiplying together the isomorphisms

$$\begin{aligned}\langle L, LM\rangle &\cong \det Rf_*(L^2M) \otimes (\det Rf_*L)^{-1} \otimes (\det Rf_*(LM))^{-1} \otimes \det Rf_*\omega_f\\ \langle L, L\rangle &\cong \langle L, L^{-1}\rangle^{-1} \cong \det Rf_*L \otimes \det Rf_*(L^{-1}) \otimes (\det Rf_*\omega_f)^{-2}\\ \langle L, M\rangle &\cong \det Rf_*(LM) \otimes (\det Rf_*L)^{-1} \otimes (\det Rf_*M)^{-1} \otimes \det Rf_*\omega_f\\ \langle L, \omega_f\rangle^{-1} &\cong \langle L^{-1}, \omega_f\rangle \cong \det Rf_*(L^{-1}\omega_f) \otimes (\det Rf_*L^{-1})^{-1}\end{aligned}$$

shows that

$$\det Rf_*(L^2M) \cong \langle L, L\rangle^2 \otimes \langle L, M\rangle^2 \otimes \langle L, \omega_f\rangle^{-1} \otimes \det Rf_*M. \tag{3.4}$$

As $f : \mathcal{X} \to B$ varies among all families of spin curves, the line bundles $\langle \zeta_f, \zeta_f\rangle$ define an ε-invariant line bundle on $\overline{\mathcal{S}}_g$. This gives rise to a class in $\mathrm{Pic}(\overline{\mathcal{S}}_g)$, which, by abuse of language, we shall indicate by $\langle \zeta, \zeta\rangle$. Similarly, one can define classes $\langle \omega, \omega\rangle$, $\langle \zeta, \mathcal{E}\rangle$, $\langle \mathcal{E}, \mathcal{E}\rangle$, and so on. The relations among these classes and the classes introduced in [1] are summarized in the following result, where we have set

$$\delta = \sum_{i\geq 0}\delta_i \qquad\qquad \vartheta = \beta_0 + \sum_{i>0}(\alpha_i + \beta_i).$$

(3.5) **Proposition**. *The following relations hold in* $\mathrm{Pic}(\overline{\mathcal{S}}_g)$*:*

i) $\langle \omega, \mathcal{E} \rangle = 0$,
ii) $\langle \zeta, \mathcal{E} \rangle = \vartheta$,
iii) $\langle \mathcal{E}, \mathcal{E} \rangle = -2\vartheta$,
iv) $\langle \zeta, \zeta \rangle = 2\lambda - 2\mu - \vartheta$,
v) $\langle \zeta, \omega \rangle = 4\lambda - 4\mu - \vartheta = -12\mu - \vartheta + \alpha_0$,
vi) $\langle \omega, \omega \rangle = 12\lambda - \delta = -24\mu - \delta + 3\alpha_0 = -24\mu - 2\vartheta + 2\alpha_0$.

We begin by proving *i)* and *ii)*. Let $f : \mathcal{X} \to B$ be a family of spin curves. Then, using the definition of $\mathcal{E}_f$ and duality we find that

$$\begin{aligned}
\langle \omega_f, \mathcal{E}_f \rangle &= \det Rf_*(\omega_f \mathcal{E}_f) \otimes (\det Rf_* \omega_f)^{-1} \otimes (\det Rf_* \mathcal{E}_f)^{-1} \otimes \det Rf_* \mathcal{O}_{\mathcal{X}} \\
&= \det Rf_* \mathcal{E}_f^{-1} \otimes (\det Rf_* \mathcal{E}_f)^{-1}, \\
\langle \zeta_f, \mathcal{E}_f \rangle &= \det Rf_*(\zeta_f \mathcal{E}_f) \otimes (\det Rf_* \zeta_f)^{-1} \otimes (\det Rf_* \mathcal{E}_f)^{-1} \otimes \det Rf_* \mathcal{O}_{\mathcal{X}} \\
&= (\det Rf_* \mathcal{E}_f)^{-1} \otimes \det Rf_* \mathcal{O}_{\mathcal{X}},
\end{aligned}$$

so $\alpha_f : \zeta_f^2 \to \omega_f$ yields canonical trivializations of $\langle \omega_f, \mathcal{E}_f \rangle$ and $\langle \zeta_f, \mathcal{E}_f \rangle$ away from the fibers of f that lie in A_i^{ev}, A_i^{odd}, $i > 0$, or in B_i^{ev}, B_i^{odd}, $i \geq 0$. Thus, both $\langle \omega, \mathcal{E} \rangle$ and $\langle \zeta, \mathcal{E} \rangle$ are integral linear combinations of boundary classes other than α_0^{ev} and α_0^{odd}. To compute the coefficients, it suffices to evaluate the degrees of $\langle \omega_f, \mathcal{E}_f \rangle$ and $\langle \zeta_f, \mathcal{E}_f \rangle$ for families $f : \mathcal{X} \to B$ of spin curves such that B is a smooth curve and the general fiber of f is smooth. For any $i > 0$ (resp., any $i \geq 0$) denote by E_i (resp., F_i) the divisor on X consisting of all exceptional components of type A_i^{ev} or A_i^{odd} (resp., B_i^{ev} or B_i^{odd}) in the fibers of f. Notice that $\mathcal{E}_f = \mathcal{O}_{\mathcal{X}}(\sum E_i + \sum F_i)$. Then, since ω_f and ζ_f restrict, respectively, to a trivial line bundle and a line bundle of degree one on each exceptional component,

$$\begin{aligned}
\deg \langle \omega_f, \mathcal{E}_f \rangle &= (\omega_f \cdot \mathcal{E}_f) = 0, \\
\deg \langle \zeta_f, \mathcal{E}_f \rangle = (\zeta_f \cdot \mathcal{E}_f) &= \sum_{i>0} \deg_f(\alpha_i) + \sum_{i \geq 0} \deg_f(\beta_i) = \deg_f \vartheta.
\end{aligned}$$

This proves *i)* and *ii)*; since $\mathcal{E} = \omega \zeta^{-2}$, *iii)* follows from them by linearity. That $\langle \omega, \omega \rangle = 12\lambda - \delta$ is due to Mumford [5]. As for *iv)*, formula (3.3), applied to $L = \zeta$, yields

$$\langle \zeta, \zeta \rangle = \langle \zeta, \omega \rangle + 2\mu - 2\lambda.$$

Since $\mathcal{E}\zeta^2 = \omega$, *ii)* and the bilinearity of $\langle\ ,\ \rangle$ imply *iv)*. Formula *iv)*, in turn, implies the following result, whose proof is the main goal of the present note.

(3.6) **Theorem**. $\alpha_0 = 4\lambda + 8\mu$.

To see this, notice that

$$\begin{aligned}
12\lambda - \delta = \langle \omega, \omega \rangle &= \langle \mathcal{E}, \omega \rangle + 2\langle \zeta, \omega \rangle \\
&= 2\langle \zeta, \mathcal{E} \rangle + 4\langle \zeta, \zeta \rangle = 2\vartheta + 8\lambda - 8\mu - 4\vartheta.
\end{aligned}$$

Noticing that it follows from (3.1) that

$$\delta = 2\vartheta + \alpha_0, \tag{3.7}$$

this proves (3.6). The remaining parts of (3.5) follow by combining (3.6), (3.7), and parts *i)*, *ii)*, *iii)*, and *iv)* of (3.5) itself.

The construction of λ and μ can be generalized as follows. Fix integers n, m and, for any family $f : \mathcal{X} \to B$ of spin curves, set

$$L_f = \det Rf_*(\zeta_f^n \mathcal{E}_f^m).$$

This defines a line bundle on $\overline{\mathcal{S}}_g$; we shall denote by $\mu_{n,m}$ the corresponding class in $\mathrm{Pic}(\overline{\mathcal{S}}_g)$. Notice that $\mu_{1,0} = \mu$ and that, by duality and because of the fact that $\omega = \mathcal{E}\zeta^2$, $\mu_{n,m}$ equals $\mu_{2-n,1-m}$.

(3.8) **Proposition**. *For any integers n and m, the following holds in* $\mathrm{Pic}(\overline{\mathcal{S}}_g)$*:*

$$\mu_{n,m} = (n^2 - 2n)(\lambda - \mu) - \frac{n^2 - n + 2m^2 - 2nm}{2}\vartheta + \lambda.$$

In view of (3.5), formula (3.4) makes it possible to calculate $\mu_{n,m}$ provided one knows how to express $\mu_{0,0}$, $\mu_{1,0}$, $\mu_{0,1}$, and $\mu_{1,1}$ in terms of λ, μ, and ϑ. Now, $\mu_{0,0}$ is just λ, $\mu_{1,0}$ equals μ, while, using duality, (3.4), and (3.5), one gets

$$\mu_{1,1} = \mu_{1,0} = \mu,$$
$$\mu_{0,1} = \mu_{2,0} = \lambda - \vartheta.$$

The remaining computations are left to the reader.

All the classes we have defined belong to the subgroup G of $\mathrm{Pic}(\overline{\mathcal{S}}_g)$ generated by the classes μ^{ev}, μ^{odd}, λ^{ev}, λ^{odd}, α_i^{ev}, α_i^{odd}, $i = 1, \dots, [\frac{g}{2}]$, β_i^{ev}, $i = 0, \dots, [\frac{g}{2}]$, and β_i^{odd}, $i = 0, \dots, [\frac{g-1}{2}]$. These generators are independent, as follows from (3.6) and the independence of the classes λ^{ev}, λ^{odd}, α_i^{ev}, α_i^{odd}, β_i^{ev}, and β_i^{odd}, proved in [1]. Thus, so far as we know, $\mathrm{Pic}(\overline{\mathcal{S}}_g)$ has no torsion. Remarkably, as was already observed in [1], Theorem (3.6) implies instead that $\lambda + 2\mu$ maps to a 4-torsion class in $\mathrm{Pic}(\mathcal{S}_g)$. More exactly, if we denote by ϱ the restriction map from $\mathrm{Pic}(\overline{\mathcal{S}}_g)$ to $\mathrm{Pic}(\mathcal{S}_g)$, we have the following result.

(3.9) **Proposition**. *The subgroup $\varrho(G)$ of* $\mathrm{Pic}(\mathcal{S}_g)$ *is the direct sum of two infinite cyclic groups generated by $\varrho(\mu^{ev})$ and $\varrho(\mu^{odd})$ and of two cyclic groups of order four generated by $\varrho(\lambda^{ev} + 2\mu^{ev})$ and $\varrho(\lambda^{odd} + 2\mu^{odd})$.*

To prove this, notice that, since all boundary classes map to zero in $\mathrm{Pic}(\mathcal{S}_g)$, $\varrho(G)$ is generated by $\varrho(\mu^{ev})$, $\varrho(\mu^{odd})$, $\varrho(\lambda^{ev} + 2\mu^{ev})$, and $\varrho(\lambda^{odd} + 2\mu^{odd})$. Moreover, (3.6) shows that

$$4\varrho(\lambda^{ev} + 2\mu^{ev}) = 4\varrho(\lambda^{odd} + 2\mu^{odd}) = 0.$$

Suppose there are integers h and k such that

$$h\varrho(\mu^{ev}) + k\varrho(\lambda^{ev} + 2\mu^{ev}) = 0.$$

Then $h\mu^{ev} + k(\lambda^{ev} + 2\mu^{ev})$ is a linear combination of even boundary classes. Using (3.6), we find that

$$h\mu^{ev} + k(\lambda^{ev} + 2\mu^{ev}) = 4l(\lambda^{ev} + 2\mu^{ev}) + \cdots,$$

where the dots stand for a linear combination of even boundary classes different from α_0^{ev}. It follows from the independence of μ^{ev}, λ^{ev}, $\alpha_i^{ev}, i > 0$, $\beta_i^{ev}, i \geq 0$, that $h = 0$ and $4 \mid k$, as desired. The argument for $\varrho(\mu^{odd})$ and $\varrho(\lambda^{odd} + 2\mu^{odd})$ is the same.

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References

1. M. Cornalba, *Moduli of curves and theta-characteristics,* in "Lectures on Riemann Surfaces" (M. Cornalba, X. Gomez-Mont, A. Verjovsky editors), World Scientific, Singapore 1989, 560-589.
2. P. Deligne, *La formule de dualité globale,* in "Théorie des Topos et Cohomologie Étale des Schémas" (S.G.A. 4), tome 3, Springer Lecture Notes in Mathematics 305, 481-587.
3. P. Deligne, *Le déterminant de la cohomologie,* in "Current Trends in Arithmetical Algebraic Geometry" (K. A. Ribet editor), Contemporary Mathematics 67, 93-177.
4. J. Harris and D. Mumford, *On the Kodaira dimension of the moduli space of curves,* Invent. Math. 67 (1982), 23-86.
5. D. Mumford, *Stability of projective varieties,* L'Enseignement Mathém. 23 (1977), 39-110.
6. D. Mumford, *Picard groups of moduli problems,* in "Arithmetical Algebraic Geometry" (O. F. G. Schilling editor), Harper & Row, New York 1965, 33-81.

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