Ex 1: $f(x) = \sqrt{3x - x^2 - 2}$

- cond. di esistenza $3x - x^2 - 2 \geq 0 \iff 1 \leq x \leq 2$

  $\mathrm{dom} f = [1,2]$

- non ci sono asintoti

- $f'(x) = \dfrac{3-2x}{2\sqrt{3x-x^2-2}}$ per $x \in]1,2[$

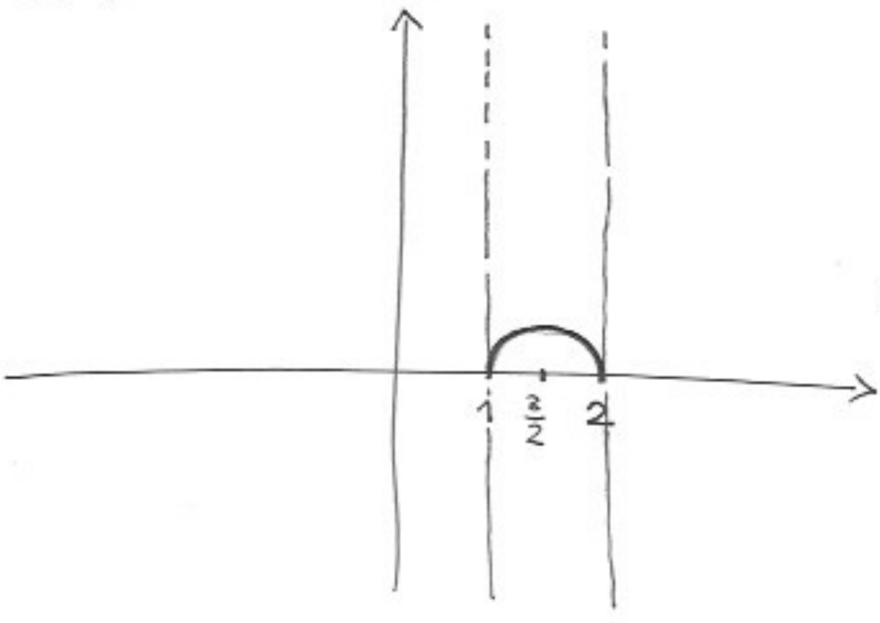
  $f'(x) \geq 0 \iff 3-2x \geq 0 \iff x < \frac{3}{2}$

  $3x - x^2 - 2 = (2-x)(x-1)$

  Per $x \to 2^-$ $\quad f(x) \sim \sqrt{2-x}$

  Per $x \to 1^+$ $\quad f(x) \sim \sqrt{x-1}$

  Dunque $x = 1$ e $x = 2$ sono punti a tangente verticale.



$x = \frac{3}{2}$ pto di max assoluto

$x = 1$ e $x = 2$ punti di minimo assoluto

Ex 2

a)
$$\frac{n^2+3n-7}{n^2+2n+1} = 1 + \frac{n-8}{n^2+2n+1}$$

$$\left(\frac{n^2+3n-7}{n^2+2n+1}\right)^{5n} = \underbrace{\left[\left(1+\frac{1}{\frac{n^2+2n+1}{n-8}}\right)^{\frac{n^2+2n+1}{n-8}}\right]}_{\downarrow\, n\to+\infty \atop e}{}^{\overbrace{\frac{5n(n-8)}{n^2+2n+1}}^{\to 5}}$$

$$\longrightarrow e^5$$

b) $\sin x \sim x$ per $x \to 0$

$$\frac{x^2}{(\log x)^2 \sin x} \sim \frac{x}{(\log x)^2} \longrightarrow \frac{0}{\infty} = 0$$

Ex 3

f è continua in $x=0$ e $f(0)=1$

$$\lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{\cos\sqrt{|x|}-1}{x} =$$

$$= \lim_{x\to 0} \frac{-\frac{1}{2}|x|}{x}$$

$\cos t \sim 1 - \frac{t^2}{2}$ per $t \to 0$

A questo punto:

$$\lim_{x\to0^+} \frac{-\frac{1}{2}x}{x} = -\frac{1}{2}$$

$$\lim_{x\to0^-} \frac{\frac{1}{2}x}{x} = \frac{1}{2}$$

$x=0$ è un punto angoloso

Ex 4

Per la conv. assoluta:

$$\left|(-1)^n \frac{1}{1+\log n}\right| = \frac{1}{1+\log n} \sim \frac{1}{\log n}$$

Ma $\log n < n$ e dunque $\frac{1}{\log n} > \frac{1}{n}$ $\forall n\geq 2$

Per confronto la serie $\sum \frac{1}{\log n}$ diverge e dunque la serie iniziale non converge assolutamente

Per la conv. semplice, si applica il CR. di Leibniz:

$\left\{\frac{1}{1+\log n}\right\}$ è infinitesima e decrescente e dunque la serie data conv. semplicemente.

Ex 5:

$$|\cos x| = \begin{cases} \cos x & \text{se } x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ -\cos x & \text{se } x \in \left[\frac{\pi}{2}, \frac{3}{2}\pi\right] \end{cases}$$

$$\int_{-\frac{\pi}{2}}^{\frac{3}{2}\pi} (3x^2+1)\,|\cos x|\,dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (3x^2+1)\cos x\,dx - \int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} (3x^2+1)\cos x\,dx$$

$$\int (3x^2+1)\cos x\,dx = (3x^2+1)\,\mathrm{sen}\,x - \int 6x\,\mathrm{sen}\,x\,dx$$

$$= (3x^2+1)\,\mathrm{sen}\,x + 6x\cos x - \int 6\cos x\,dx$$

$$= (3x^2+1)\,\mathrm{sen}\,x + 6x\cos x - 6\,\mathrm{sen}\,x + c$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (3x^2+1)\cos x\,dx = \Big[(3x^2+1)\,\mathrm{sen}\,x + 6x\cos x - 6\,\mathrm{sen}\,x\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \left(\frac{3}{4}\pi^2 + 1 - 6\right)\cdot 2 = \frac{3}{2}\pi^2 - 10$$

$$\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} (3x^2+1)\cos x\,dx = \Big[(3x^2+1)\,\mathrm{sen}\,x + 6x\cos x - 6\,\mathrm{sen}\,x\Big]_{\frac{\pi}{2}}^{\frac{3}{2}\pi} =$$

$$= -\left(\frac{27}{4}\pi^2 + 1\right) + 6 - \left\{\left(\frac{3}{4}\pi^2 + 1\right) - 6\right\}$$

$$= -\frac{15}{2}\pi^2 + 10$$

L'integrale assegnato è pari a $\frac{3}{2}\pi^2 - 10 + \frac{15}{2}\pi^2 - 10 =$

$$9\pi^2 - 20$$

Ex 6 :

- $1-i = \sqrt{2}\left(\cos\frac{7}{4}\pi + i\,\text{sen}\,\frac{7}{4}\pi\right)$

$$(1-i)^8 = (\sqrt{2})^8\left(\cos 14\pi + i\,\text{sen}\,14\pi\right) = 16$$

- $\dfrac{(1+i)^2}{3+4i} = \dfrac{\cancel{1}-\cancel{1}+2i}{3+4i}\cdot\dfrac{3-4i}{3-4i} = \dfrac{6i+8}{25} = \dfrac{8}{25}+\dfrac{6}{25}i$

- $\left(\dfrac{1-i}{i}\right)^7 = \left[\dfrac{\sqrt{2}\left(\cos\frac{7}{4}\pi + i\,\text{sen}\,\frac{7}{4}\pi\right)}{\cos\frac{\pi}{2}+i\,\text{sen}\,\frac{\pi}{2}}\right]^7 =$

$$= (\sqrt{2})^7\left\{\cos\left[7\left(\frac{7}{4}\pi-\frac{\pi}{2}\right)\right] + i\,\text{sen}\left[7\left(\frac{7}{4}\pi-\frac{\pi}{2}\right)\right]\right\}$$

$$= 8\sqrt{2}\left(\cos\frac{35}{4}\pi + i\,\text{sen}\,\frac{35}{4}\pi\right) =$$

$$\frac{35}{4} = 8 + \frac{3}{4}$$

$$= 8\sqrt{2}\left(\cos\frac{3}{4}\pi + i\,\text{sen}\,\frac{3}{4}\pi\right) = -8+8i$$

- $\dfrac{1+2i}{3-i} + \dfrac{2-i}{5i} = \dfrac{(1+2i)(3+i)}{10} - \dfrac{(2-i)\;\;i}{5} = \dfrac{1+7i-4i-2}{10}$

$$= -\frac{1}{10}+\frac{3}{10}i$$