1) $f(x) = \operatorname{arctg} x - \dfrac{x+1}{x^2+1}$

$\mathrm{Dom}\, f = \mathbb{R}$

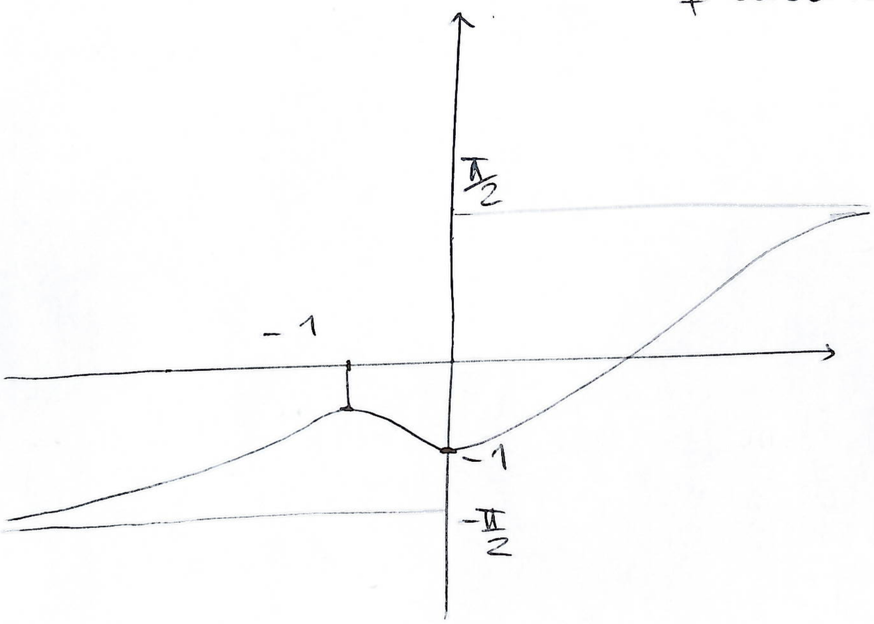
$\lim\limits_{x \to +\infty} f(x) = \frac{\pi}{2}$ $\quad y = \frac{\pi}{2}$ asint. orizz. a $+\infty$

$\lim\limits_{x \to -\infty} f(x) = -\frac{\pi}{2}$ $\quad y = -\frac{\pi}{2}$ asint. orizz. a $-\infty$

$f'(x) = \dfrac{2x(x+1)}{(x^2+1)^2} \geq 0 \iff x \leq -1 \vee x \geq 0$

f crescente in $]-\infty, -1] \cup [0, +\infty[$

f decresc. in $[-1, 0]$



$f(0) = -1$

$x = -1$ pto di max loc.

$x = 0$ pto di min loc.

2) a) $(n-1)! + 3^n \sim (n-1)!$ per $n \to +\infty$

$\operatorname{sen}\left(\frac{2}{n}\right) \sim \frac{2}{n}$ per $n \to +\infty$

$$\frac{n!\, \operatorname{sen}\left(\frac{2}{n}\right) \operatorname{arctg} n}{(n-1)! + 3^n} \sim \frac{\overbrace{n!}^{n(n-1)!} \frac{2}{n} \cdot \operatorname{arctg} n}{(n-1)!} = 2 \operatorname{arctg} n \xrightarrow[n \to +\infty]{} \pi$$

b) $\lim_{x\to 2} \dfrac{\sqrt[3]{4x}\ \operatorname{sen}^2(x-2)}{(x+1)^2 \log^2(x-1)} = \dfrac{2}{9} \lim_{x\to 2} \dfrac{\operatorname{sen}^2(x-2)}{\log^2(x-1)} =$

$$\underset{t=x-2}{=} \frac{2}{9} \lim_{t\to 0} \frac{\operatorname{sen}^2 t}{\log^2(t+1)} = \frac{2}{9} \lim_{t\to 0} \frac{\cancel{t^2}}{\cancel{t^2}} = \frac{2}{9}$$

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o stime asintotiche

3) $\lim_{x\to 0} e^{-\frac{1}{x^2}} = 0 \Rightarrow$ f si prolunga con continuità in $x=0$ ponendo $f(0)=0$

$$f'(x) = \frac{2}{x^3} e^{-\frac{1}{x^2}}$$

$$\lim_{x\to 0} f'(x) \underset{t=\frac{1}{x}}{=} \lim_{t\to\pm\infty} 2t^3 e^{-t^2} = \lim_{t\to\pm\infty} \frac{2t^3}{e^{t^2}} = 0$$

$\Rightarrow \exists\ f'(0) = 0$

$$f''(x) = -\frac{6}{x^4} e^{-\frac{1}{x^2}} + \frac{2}{x^3} e^{-\frac{1}{x^2}} \cdot \frac{2}{x^3} = e^{-\frac{1}{x^2}} \left(-\frac{6}{x^4} + \frac{4}{x^6}\right)$$

$$\lim_{x\to 0} f''(x) = \lim_{x\to 0} -\frac{6}{x^4} e^{-\frac{1}{x^2}} + \lim_{x\to 0} \frac{4}{x^6} e^{-\frac{1}{x^2}} = 0$$

$\Rightarrow \exists\ f''(0) = 0$ e così via $f^{(k)}(0) = 0\ \forall k$

4) $\boxed{\alpha = 1}$

$$\left| \frac{(-1)^n}{n} \log\left(1+\frac{1}{\sqrt{n}}\right) \right| = \frac{1}{n} \cdot \log\left(1+\frac{1}{\sqrt{n}}\right) \sim \frac{1}{n} \cdot \frac{1}{\sqrt{n}} = \frac{1}{n^{3/2}}$$

La serie $\sum_1 \frac{1}{n^{3/2}}$ converge; quindi anche

$\sum_1 \frac{1}{n} \log\left(1+\frac{1}{\sqrt{n}}\right)$ converge (confr. asint.)

La serie data conv. assolut. e quindi anche semplic.

$\boxed{\alpha = \frac{1}{2}}$

$$\left| \frac{(-1)^n}{\sqrt{n}} \log\left(1+\frac{1}{\sqrt{n}}\right) \right| = \frac{1}{\sqrt{n}} \log\left(1+\frac{1}{\sqrt{n}}\right) \sim \frac{1}{n}$$

La serie armonica $\sum_1 \frac{1}{n}$ diverge, dunque la serie data non conv. assolutamente.

La successione $\frac{1}{\sqrt{n}} \log\left(1+\frac{1}{\sqrt{n}}\right)$ è infinitesima e decrescente (prodotto di due succ. positive e decrescenti)

Per il CR. di Leibnitz la serie data conv. semplic.

5) $f(x)=\dfrac{3x+1}{x+\sqrt{x}}$ è positiva e continua in $(0,1]$

$\lim_{x\to 0^+} f(x)=+\infty$ l'integrale deve essere inteso in senso generalizzato

$f(x) \sim \dfrac{1}{\sqrt{x}}$ per $x\to 0^+$ e $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\,dx$ converge

$\Rightarrow$ $\displaystyle\int_0^1 f(x)\,dx$ converge per il cr. di confronto asintotico

$t=\sqrt{x} \Rightarrow x=t^2 \Rightarrow dx=2t\,dt$

$$\int_0^1 \frac{3t^2+1}{t^2+t}\cdot 2t\,dt = 2\int_0^1 \frac{3t^2+1}{t+1}\,dt \circledcirc$$

$3t^2+1=3(t+1)(t-1)+4$ (divisione tra polinomi)

$$\frac{3t^2+1}{t+1}=3(t-1)+\frac{4}{t+1}$$

$$\circledcirc\ 6\int_0^1 (t-1)\,dt + 8\int_0^1 \frac{1}{t+1}\,dt =$$

$$6\left[\frac{t^2}{2}-t\right]_0^1 + 8\Big[\log|t+1|\Big]_0^1 = -3+8\log 2$$

1) $\left(z^3 - \dfrac{1+i}{1-i}\right)\left(z^2 - |z|\right) = 0$

$z^3 - \dfrac{1+i}{1-i} = 0 \quad \vee \quad z^2 - |z| = 0$

$z^3 = \dfrac{1+i}{1-i} = \dfrac{(1+i)^2}{2} = i$

$= \left(\cos\frac{\pi}{2} + i \operatorname{sen}\frac{\pi}{2}\right)$

$z_1 = \left(\cos\frac{\pi}{6} + i \operatorname{sen}\frac{\pi}{6}\right)$

$z_2 = \left(\cos\frac{5}{6}\pi + i \operatorname{sen}\frac{5}{6}\pi\right)$

$z_3 = \left(\cos\frac{3}{2}\pi + i \operatorname{sen}\frac{3}{2}\pi\right)$

$z^2 = |z|$

$\rho^2(\cos 2\theta + i \operatorname{sen} 2\theta) = \rho(\cos 0 + i \operatorname{sen} 0)$

$$\begin{cases} \rho^2 = \rho \\ 2\theta = 0 + 2k\pi \end{cases}$$

$$\begin{cases} \rho = 0 ; \rho = 1 \\ \theta = k\pi \end{cases}$$

$z_4 = 0 \quad (\rho = 0)$

$z_5 = 1 \quad (\rho = 1, k = 0)$

$z_6 = -1 \quad (\rho = 1, k = 1)$