

Esercizio 1

$$f(x,y) = \frac{1}{2}x^2y^2 - 2y^2 + \frac{1}{3}x^3$$

$$\frac{\partial f}{\partial x} = xy^2 + x^2$$

$$\frac{\partial f}{\partial y} = x^2y - 4y$$

$$\nabla f = 0 \Leftrightarrow \begin{cases} xy^2 + x^2 = 0 \\ x^2y - 4y = 0 \end{cases} \Leftrightarrow \begin{cases} x(y^2 + x) = 0 \\ y(x^2 - 4) = 0 \end{cases}$$

$$\begin{cases} x=0 \\ y=0 \end{cases} \vee \begin{cases} y^2 + 2 = 0 \text{ imp.} \\ x=2 \end{cases} \vee \begin{cases} y = \pm\sqrt{2} \\ x = -2 \end{cases}$$

$P_1(0,0)$ $P_2(-2, \sqrt{2})$ $P_3(-2, -\sqrt{2})$ punti stazionari

$$Hf(x,y) = \begin{pmatrix} y^2 + 2x & 2xy \\ 2xy & x^2 - 4 \end{pmatrix}$$

$$Hf(0,0) = \begin{pmatrix} 0 & 0 \\ 0 & -4 \end{pmatrix} \quad \text{semid. neg.}$$

$$Hf(-2, \sqrt{2}) = \begin{pmatrix} -2 & -4\sqrt{2} \\ -4\sqrt{2} & 0 \end{pmatrix} \quad \det < 0 \Rightarrow \text{matrice indefinita}$$

$\Rightarrow P_2$ sella

$Hf(P_3)$ indif. $\Rightarrow P_3$ sella

Per P_1 non posso dire nulla. Ma guardando alla restrizione $f(x,0) = \frac{1}{3}x^3$, concludo che

anche P è punto di sella.

Esercizio 2

$$a_n = \frac{2^n (n+1)}{n^2+3}$$

$$\sqrt[n]{a_n} = \frac{2 \sqrt[n]{n+1}}{\sqrt[n]{n^2+3}} \rightarrow 2 \Rightarrow R = \frac{1}{2}$$

$$x = \frac{1}{2} \quad \sum_{n=1}^{+\infty} \frac{2^n (n+1)}{n^2+3} \cdot \frac{1}{2^n}$$

$$\frac{n+1}{n^2+3} \sim \frac{1}{n}$$

\Rightarrow la serie diverge
CR, confronto
asint.

$$x = -\frac{1}{2} \quad \sum_{n=1}^{+\infty} (-1)^n \frac{n+1}{n^2+3}$$

converge per il CR.
di Leibniz

$\left(b_n = \frac{n+1}{n^2+3} \text{ è infinitesima, decrescente} \right)$

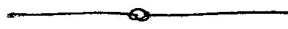
Esercizio 3

$$0 \leq |f(x,y)| \leq \frac{|x^2 y| + |3y^3|}{x^2 + y^2} = \underbrace{\frac{x^2}{x^2 + y^2}}_{\text{limitata}} \cdot \underbrace{|y|}_{\text{infinitesima}} + \frac{\overbrace{3y^2}^{\text{limitata}}}{\underbrace{x^2 + y^2}_{\text{infinitesima}}} \cdot \underbrace{|y|}_{\text{infinitesima}} \rightarrow 0$$

Per il CR. del confronto anche $\lim_{(x,y) \rightarrow (0,0)} f(x,y) = 0$
 da cui f continua in $(0,0)$.

Oppure :

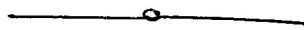
$$\begin{aligned}
 |f(\rho \cos \theta, \rho \sin \theta)| &= |\rho \cos^2 \theta \sin \theta - 3 \rho \sin^3 \theta| = \\
 &= \rho |\cos^2 \theta \sin \theta - 3 \sin^3 \theta| \leq \\
 &\leq \rho (|\cos^2 \theta \sin \theta| + 3|\sin^3 \theta|) \\
 &\leq 4\rho \xrightarrow{\rho \rightarrow 0} 0
 \end{aligned}$$



$$\frac{\partial f}{\partial x}(0,0) = \frac{d}{dx} f(x,0) \Big|_{x=0} = \frac{d}{dx} 0 \Big|_{x=0} = 0$$

$$\frac{\partial f}{\partial y}(0,0) = \frac{d}{dy} f(0,y) \Big|_{y=0} = \frac{d}{dy} (-3y) \Big|_{y=0} = -3$$

$$\Rightarrow \nabla f(0,0) = (0, -3)$$



$$f \text{ diff.}^a \text{ in } (0,0) \Leftrightarrow \lim_{(x,y) \rightarrow (0,0)} \frac{\frac{x^2 y - 3y^3}{x^2 + y^2} + 3y}{\sqrt{x^2 + y^2}} = 0$$

$$\Leftrightarrow \lim_{(x,y) \rightarrow (0,0)} \frac{4x^2 y}{(x^2 + y^2)^{3/2}} = 0$$

Ma in coord. polari $\xrightarrow{\quad} = 4 \cos^2 \theta \sin \theta \rightarrow 0$

Pertanto f non è differenziabile in $(0,0)$.

ESERCIZIO 4

$$\gamma(t) = (\cos t, \sin t, e^{2t}) \quad t \in [0, 2\pi]$$

γ è di classe C^1 e $\gamma'(t) = (-\sin t, \cos t, 2e^{2t}) \neq (0, 0, 0)$

$\forall t \in [0, 2\pi] \Rightarrow \gamma$ è regolare

$\gamma(0) \neq \gamma(2\pi) \Rightarrow \gamma$ non è chiusa

$$\begin{aligned} \int_{\gamma} z^2 ds &= \int_0^{2\pi} e^{4t} \sqrt{1 + 4e^{4t}} dt = \frac{(1 + 4e^{4t})^{3/2}}{\frac{3}{2} \cdot 16} \Big|_0^{2\pi} = \\ &= \frac{1}{24} \left[(1 + 4e^{8\pi})^{3/2} - 5^{3/2} \right] \end{aligned}$$

$$\begin{aligned} \int_{\gamma} \vec{F} \cdot d\vec{r} &= \int_0^{2\pi} (e^{2t} \cos t, e^{2t} \sin t, \sin t) \cdot (-\sin t, \cos t, 2e^{2t}) dt \\ &= \int_0^{2\pi} 2e^{2t} \sin t dt = I \end{aligned}$$

Integrando per parti: due volte:

$$\begin{aligned} I &= -2e^{2t} \cos t \Big|_0^{2\pi} + \int_0^{2\pi} 4e^{2t} \cos t dt = -2e^{2t} \cos t \Big|_0^{2\pi} + 4e^{2t} \sin t \Big|_0^{2\pi} - \int_0^{2\pi} 8e^{2t} \sin t dt \\ &= -2e^{4\pi} + 2 - 4I \end{aligned}$$

$$\Rightarrow I = \frac{2}{5} (1 - e^{4\pi})$$

ESERCIZIO 5

La normale richiesta $\hat{n} = \frac{(2_x f, 2_y f, -1)}{\sqrt{1 + |\nabla f|^2}}$

dove $f(x, y) = \sqrt{x^2 + y^2}$.

Quindi

$$\hat{n} = \frac{\left(\frac{x}{\sqrt{x^2 + y^2}}, \frac{y}{\sqrt{x^2 + y^2}}, -1 \right)}{\sqrt{1 + |\nabla f|^2}}$$

$$\text{Flusso} = \iint_D (x, x^2 + y^2, y^2 \sqrt{x^2 + y^2}) \cdot \left(\frac{x}{\sqrt{x^2 + y^2}}, \frac{y}{\sqrt{x^2 + y^2}}, -1 \right) dx dy$$

$$D = \{ 1 < x^2 + y^2 < 4 \}$$

$$= \iint_D \left(\frac{x^2}{\sqrt{x^2 + y^2}} + y \sqrt{x^2 + y^2} - y^2 \sqrt{x^2 + y^2} \right) dx dy =$$

$$= \int_0^{2\pi} \int_1^2 (p^2 \cos^2 \theta + p^3 \sin \theta - p^4 \sin^2 \theta) dp d\theta$$

$$= \frac{1}{3} \cdot 7 \cdot \pi + 0 - \frac{31}{5} \cdot \pi = -\frac{58}{15} \pi$$

Esercizio 6

$$f(x,y) = \frac{|x^3 y|}{x^2 + y^2} \quad \bar{e} \text{ pari in } x \text{ e in } y$$

Pertanto, essendo D simmetrico rispetto ad entrambi gli assi, abbiamo

$$\iint_D \frac{|x^3 y|}{x^2 + y^2} dx dy = 4 \iint_{D^{++}} \frac{x^3 y}{x^2 + y^2} dx dy$$

$$D^{++} = D \cap \{x > 0\} \cap \{y > 0\}$$

$$= 4 \int_0^{\frac{\pi}{2}} \int_1^3 \rho^3 \cos^3 \theta \sin \theta d\rho d\theta =$$

$$= \cancel{4} \cdot \frac{80}{\cancel{4}} \cdot \left(-\frac{\cos^4 \theta}{4} \right) \Big|_0^{\frac{\pi}{2}} = 20$$