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DIMO2013 – Diffuse interface models

Levico Terme (TN) – September 10–13, 2013

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BOOK OF ABSTRACTS / TALKS AND POSTERS

TALKS

Sharp interface limit for the Cahn-Larche system

HELMUT ABELS

University of Regensburg (Germany)

The Cahn-Larche system is a system, which couples the Cahn-Hilliard equation with the system of linearized elasticity. It is derived in order to describe anisotropies in the material due a crystaline structure, which can play an important role during the phase separation. We discuss the sharp interface limit of this diffuse interface model to its sharp interface limit, where the Mullins-Sekerka equation is coupled to the system of linearized elasticity. As long as the limit system possesses a classical smooth solution convergence is proved by adapting the method of Alikakos, Bates and Chen for the single Cahn-Hilliard equation. To this end a finite piece of the expansion, derived by the formally matched asymptotics calculations, is constructed and its difference to the exact solution is estimated. This is a joint-work with Stefan Schaubeck.

On some nonlinear parabolic equation involving variable exponents

GORO AKAGI

University of Kobe (Japan)

This talk is concerned with the following doubly nonlinear parabolic equation involving variable exponents:

$$|\partial_t u|^{p(x)-2}\partial_t u - \Delta_{m(x)} u = f(x,t) \quad \text{in } Q := \Omega \times (0,T), \tag{0.1}$$
$$u = 0 \quad \text{on } \partial\Omega \times (0,T), \tag{0.2}$$
$$u(\cdot,0) = u_0 \quad \text{in } \Omega, \tag{0.3}$$

where $1 < p(x), m(x) < \infty$ are variable exponents and $\Delta_{m(x)}$ stands for the $m(x)$-*Laplacian* given by

$$\Delta_{m(x)} u = \operatorname{div}\left(|\nabla u|^{m(x)-2}\nabla u\right).$$

The constant exponent case, $p(x) \equiv p$, $m(x) \equiv m$, can be treated under appropriate assumptions by using the abstract framework established by Colli in [1] (see also Colli-Visintin [2] for a Hilbert framework) for doubly nonlinear evolution equations in Banach spaces V of the form

$$A(u'(t)) + \partial\varphi(u(t)) \ni f(t) \quad \text{in } V^*, \quad 0 < t < T,$$

where $A : V \to V^*$ is a maximal monotone operator satisfying a p-growth condition and $\partial\varphi : V \to V^*$ denotes the subdifferential operator of a proper, lower semicontinuous, convex function $\varphi : V \to (-\infty, \infty]$ with precompact sublevels.

In this talk, we first exhibit new difficulties arising from the variable exponent setting, and then, discuss the existence and regularity of solutions. To prove the existence of solutions, we introduce a modified frame of maximal monotone and subdifferential operators in order to adapt it to the variable exponent setting.

This talk is based on a joint work with Giulio Schimperna (Pavia University, Italy).

[1] Colli, P., On some doubly nonlinear evolution equations in Banach spaces, Japan J. Indust. Appl. Math. **9** (1992), 181–203.

[2] Colli, P. and Visintin, A., On a class of doubly nonlinear evolution equations, Comm. Partial Differential Equations **15** (1990), 737–756.

New modeling, analysis, and computations for thermal retraction in semi-crystalline shape memory polymers

Elena Bonetti

University of Pavia (Italy)

We introduce a new model describing thermal retraction in semi-crystalline polymers. This kind of effect may be considered as a shape memory property as, after permanent deformation by mechanical loads, polymers may recover their original shape just by thermal actions. We model the phenomenon by use of a phase transition model, combining phase transformation and phase separation dynamics, as well as non-smooth internal constraints on the variables. Existence result for a suitable weak formulation of the equations (including hysteresis loops) is given as well as some numerical simluations, fitting with experimental data. The results have been obtained in collaboration with L. Castellani, M. Pachera, F. Scavello (Versalis, Basic Chemicals & Plastics Research Center, Mantova) and F. Auricchio, G. Scalet (University of Pavia).

Long-time dynamics of a hyperbolic non-isothermal viscous Cahn Hilliard equation with dynamic boundary conditions

Cecilia Cavaterra

University of Milano (Italy)

We consider an initial and boundary value problem for a non-isothermal viscous Cahn Hilliard equation in the case of dynamic boundary conditions. We prove the existence of an energy solution and of the global attractor. The long time behavior of single solution trajectories is also investigated.

Special functions with bounded deformation

Gianni Dal Maso

SISSA, Trieste (Italy)

A summary will be delivered at the beginning of the talk.

Parabolic equations on evolving domains

Charlie Elliott

University of Warwick (UK)

A summary will be delivered at the beginning of the talk.

On the weak solutions to the Euler-Fourier system

Eduard Feireisl

Academy of Sciences of the Czech Republic, Prague (Czech Republic)

We discuss several concepts of weak solutions for the system of equations describing the motion of an inviscid but still heat conducting gas (Euler-Fourier system). We show that the class of "standard" weak solutions is too large to guarantee well posedness. On the other hand, even imposing the constraint of the Second law in the form of entropy inequality is not sufficient for obtaining unique solutions. Finally, imposing also the First law as the total energy balance, we address the same issues for the resulting augmented system.

Collisions and shape memory alloys

Michel Frémond

University of Rome "Tor Vergata" (Italy)

When a solid made of shape memory alloy collides with an obstacle, the velocities of the solid adapt very rapidly to the kinematic constraint due to the obstacle. The duration of this phenomenon is short compared to the duration of the overall evolution of the structure. Thus we assume it is instantaneous and it results that the velocities are discontinuous with respect to time. Moreover, the temperature and the composition of the alloy are also discontinuous due to the dissipative character of the collision.

The principle of virtual work and the laws of thermodynamics provide the equation of motion and the constitutive law giving the velocities, the temperature and the composition of the alloy after the collision depending on the velocities, the temperature and the composition before the collision.

The resulting equations are non linear elliptic partial differential equations.

The simulations agree well with basic mechanical results and, therefore, allow to design structures made of shape memory alloys with the characteristics of dissipating the collision's energy and of coming back to their original shape after collision.

A PDE system related to solid-solid phase transition in shape memory alloys

Gianni Gilardi

University of Pavia (Italy)

A model for solid-solid phase transition in shape memory alloys has been recently introduced by Mauro Fabrizio et al. The physical quantities that are involved are the absolute temperature, the order parameter, and the scalar displacement along with the associated stress vector. In the corresponding PDE system, two parabolic equations and a second order hyperbolic equation are coupled. As the problem is highly nonlinear, even the existence of a solution is not clear at all, at first sight. The present talk provides the outline of an existence result recently obtained by the speaker in collaboration with Elena Bonetti, Pierluigi Colli and Mauro Fabrizio.

Nonlocal Cahn-Hilliard-Navier-Stokes systems with nonconstant mobility

Maurizio Grasselli

Politecnico di Milano (Italy)

We consider a diffuse interface model for incompressible isothermal mixtures of two immiscible fluids with matched densities. This model consists of the Navier-Stokes system coupled with a convective nonlocal Cahn-Hilliard equation with nonconstant and (possibly degenerate) mobility. We discuss the existence of a global weak solution and, in dimension two, the existence of a global attractor.

On convergent schemes for diffuse-interface models for two-phase flow of incompressible fluids with different mass densities

Günther Grün

University of Erlangen-Nürnberg (Germany)

In this talk, we will be concerned with the numerics of two-phase flow of immiscible, incompressible viscous fluids with different mass densities. In contrast to the case of identical mass densities, for general mass densities it has only recently been possible to formulate a diffuse interface model which is at the same time consistent with thermodynamics and which allows for a solenoidal velocity field (see Abels, Garcke, Grün M3AS2012). It consists of a new momentum equation for the velocity field coupled to a Cahn-Hilliard equation for the evolution of the order parameter. We will introduce a finite-element scheme for this model. Due to a subtle discretization of the convective coupling between the flux of the phase-field and the momentum equation, it satisfies a discrete counterpart of the natural energy estimate known from the continous setting. For this scheme, we prove convergence in two and in three space dimensions. Some key ingredients are higher regularity results for discrete solutions of convective Cahn-Hilliard equations and discrete versions of Sobolev's imbedding result. Finally, we shall present numerical simulations to underline the full practicality of our approach and to identify physical settings for which the new coupling term suggested in (Abels, Garcke, Grün, M3AS 2012) seems to be indispensable for numerical stability.

(partially joint work with Fabian Klingbeil, Erlangen)

On a initial-boundary Q-Tensor problem related to Liquid Crystals

F. Guillén-González

University of Sevilla (Spain)

In this talk, a coupled Navier-Stokes and Q-Tensor system will be studied in a bounded three-dimensional domain imposing (homogeneous) Dirichlet boundary conditions for the velocity u and several boundary conditions for the tensor Q: Dirichlet, Neumann or periodic one. The corresponding initial-value problem defined in the whole space R^3 (without boundary conditions) has been already analyzed in [Paicu and Zarnescu(2012), Paicu and Zarnescu(2011)]. Firstly, the existence of global in time weak solutions will be presented, identifying how to change the model in order to get constraints of symmetry and traceless for Q, [Guillén-González and Rodríguez-Bellido (2013a)]. Afterwards, we will discussed the problem to get local in time regular- ity for any regular data when non periodic boundary conditions are consid- ered, giving some results of local in time partial regularity and uniqueness, [Guillén-González and Rodríguez-Bellido (2013b)]. In all these results, the presence of a stretching term in the Q-system plays a crucial role.

Formal asymptotic limit of a diffuse-interface tumor-growth model

Danielle Hilhorst

University of Paris Sud (France)

We consider a family of tumor-growth models which enjoy a gradient flow structure and present a formal derivation of their asymptotic limit as a small parameter governing the interface width tends to zero. This is joint work with Johannes Kampmann, Thanh Nam Nguyen and Kris van der Zee.

Phase-field approach to models of regional economic trend

Nobuyuki Kenmochi

Bukkyo University, Kyoto (Japan)

It is said that in our future the economic growth mainly relies on the knowledge and technology development for production systems. Its dynamics is controled by various artificial ways. Therefore, it is not so easy to grasp the trend of economic quantities (capital, technological level and labour force) from the theoretical point of views. However, it seems that some prototype economic trend models are able to be derived from suitable energy functionals, when any artificial treatment is neglected in the process.

We propose an (economic) energy functional of the following type:

$$E(w, A) := \int_\Omega \left\{ \frac{\nu}{2}|\nabla w|^2 + \frac{b}{2}w^2 - \frac{\gamma(A)}{1+\alpha}w^{1+\alpha} \right\} dx + I_K(A) + \hat{f}(A),$$

where Ω is a bounded smooth domain in $\mathbf{R}^2$, α is a constant with $0 < \alpha < 1$, ν, b are positive constants, $\gamma(\cdot)$ and $\hat{f}(\cdot)$ are non-negative smooth functions on $\mathbf{R}$, K is a compact interval in $\mathbf{R}$ and $I_K(\cdot)$ is the indicator function of K with its subdifferential $\partial I_K(\cdot)$ in $\mathbf{R}$; $w = w(x,t)$ is the capital of the region Ω at time t and $A := A(t)$ is a function indicating knowledge and technological level; moreover we assume that the labor force is constant, and neglect it in the energy functional. The kinetic equations, which are derived from the above energy functional, are of the form:

$$w_t - \nu\Delta w + bw = \gamma(A)w^\alpha, \quad w(x,0) = w_0(x) > 0, \quad t > 0,$$

$$A' + \partial I_K(A) + f(A) \ni \frac{\gamma'(A)}{1+\alpha}\int_\Omega w^{1+\alpha}dx, \quad A(0) = A_0 > 0, \quad t > 0,$$

where $f(A) := \hat{f}'(A)$.

In this talk, we discuss a model of economic trend, including an artificial treatment to bring about the knowledge and technological development, for instance, the second equation of the above system replaced by

$$A' + \partial I_{K(t,\int_0^t\int_\Omega wdxd\tau)}(A) + f(A) \ni \frac{\gamma'(A)}{1+\alpha}\int_\Omega w^{1+\alpha}dx;$$

here $K(\cdot,\cdot)$ is a set-valued mapping from $[0,\infty) \times \mathbf{R}$ into the set of all compact intervals in $\mathbf{R}$ and $K(t, \int_0^t\int_\Omega wdxd\tau)$ is interpreted as the field of knowledge and technology built up by the total investment $\int_0^t\int_\Omega w(\tau,x)dxd\tau$ until time t.

Numerical solution of vector-valued phase field models

Ralf Kornhuber

Free University of Berlin (Germany)

Vector-valued phase field models play a prominent role in many practical applications such as grain boundary motion (non-conserved order parame- ter) or phase separation of multicomponent alloys. While the analysis and numerical analysis of vector-valued phase field equations already is in a good sha- pe, the fast solution of the resulting discrete algebraic systems is still in its infancy. We present novel multigrid methods for Allen-Cahn equations and nonsmooth Schur-Newton methods for Cahn-Hilliard equations with loga- rithmic potential. Global convergence is based on convex minimization rather than smoothness which leads to robustness with respect to temperature as ranging from the shallow quench to the deep quench limit. Our theoretical considerations are illustrated by numerical experiments.

Damage and phase separation processes. Modeling and analysis of nonlinear PDE-systems

CHRISTIANE KRAUS

Weierstrass Institute, Berlin (Germany)

Materials like alloys, which enable the functionality of technical products, change the microstructure over time. For instance, phase separation, coarsening, and damage processes take place.

The Cahn-Hilliard system is a well established model for describing phase separation and coarsening in alloys. To account for elastic effects, the Cahn-Hilliard system is coupled with an elliptic equation for the deformation field, the so-called Cahn-Larché system.

In this talk, we introduce a system where the Cahn-Larché model is coupled with a damage phase field. The coupling takes place in the elastic energy density of the system, which now depends on the strain, the chemical concentration and the damage variable. We assume that damage influences its local surrounding. In the case of complete damage, the system may degenerate.

The evolution of the system is described by a degenerating parabolic differential equation of fourth order for the concentration, a doubly nonlinear differential inclusion for the damage process and a degenerating quasi-static balance equation for the displacement field. All these equations are strongly nonlinearly coupled.

We present a suitable notion of weak solutions for the introduced systems and provide existence and regularity results under appropriate growth conditions for the free energy.

This is a joint work with C. Heinemann (WIAS, Berlin) and with Elena Bonetti (University of Pavia) and Antonio Segatti (University of Pavia).

Kinetic density functional theory: a time dependent hydrodynamic density functional theory for freezing

JOHN LOWENGRUB

University of California, Irvine (USA)

Understanding solid liquid phase transitions is of great importance in many applications ranging from growth of nanocrystals in solutions for solar cells to making the perfect ice cream. Classical Density Functional Theory (CDFT) has been very successful in predicting the phase transition of pair potential fluids. However the CDFT only characterizes the equilibrium of the system and does not shed light on the time evolution of the system. This talk will present an approach starting with the Revised Enskog Kinetic Theory (RET) as a definition of time evolution to develop a nonlocal hydrodynamic model for freezing of a hard sphere liquid. The relation between Kinetic Theory, CDFT and Phase Field Crystal (PFC) models will be outlined. Numerical results characterizing the time evolution of the model and its applications, including a numerical study of the effect of flow on growth and morphological evolution of nanocrystal nucleates, will be presented. This is joint work with Arvind Baskaran (UCI) and Aparna Baskaran (Brandeis).

Existence of the free boundary in a diffusive flow in porous media

Gabriela Marinoschi

Romanian Academy, Bucarest (Romania)

We provide the existence of a diffusive interface between the two phases formed during a fluid flow in porous media. The phenomenon is described by a nonlinear diffusion equation with a multivalued time-dependent nonlinearity derived from a convex potential. Existence is proved for a general potential and additional properties of the solution are discussed in the case of a stronger time regularity of the nonlinearity.

The Penrose-Fife phase-field system with dynamic boundary conditions

Alain Miranville

University of Poitiers (France)

Our aim in this talk is to discuss the well-posedness and longtime behavior of the Penrose-Fife system in phase transition with dynamic boundary conditions.

Global solvability of some double-diffusive convection systems

Mitsuharu Ôtani

Waseda University, Tokyo (Japan)

In this talk, the global solvability of the initial boundary value problem and the periodic problem are discussed for the following double-diffusive convection systems under the homogeneous Neumann boundary condition in a bounded domain Ω.

$$\text{(BF)}\begin{cases} \partial_t \boldsymbol{u} = \nu\,\Delta\boldsymbol{u} - a\,\boldsymbol{u} - \nabla p + \boldsymbol{g}\,T + \boldsymbol{h}\,C + \boldsymbol{f}_1 & (x,t)\in\Omega\times[0,S],\\ \partial_t T + \boldsymbol{u}\cdot\nabla T = \Delta T + f_2 & (x,t)\in\Omega\times[0,S],\\ \partial_t C + \boldsymbol{u}\cdot\nabla C = \Delta C + \rho\,\Delta T + f_3 & (x,t)\in\Omega\times[0,S],\\ \nabla\cdot\boldsymbol{u} = 0 & (x,t)\in\Omega\times[0,S],\\ \boldsymbol{u} = 0,\ \dfrac{\partial T}{\partial n} = 0,\ \dfrac{\partial C}{\partial n} = 0, & (x,t)\in\partial\Omega\times[0,S], \end{cases}$$

where n denotes the unit outward normal vector on $\partial\Omega$ and $\boldsymbol{u}, T, C, p$ are unknown functions and represent the solenoidal velocity of the fluid, the temperature of the fluid, the concentration of a solute, the pressure of the fluid respectively. Given constant vectors $\boldsymbol{g}, \boldsymbol{h}$ are derived from the gravity. The positive constants ρ, a are called the Soret's coefficient and Darcy's coefficient respectively and $\boldsymbol{f}_1, f_2, f_3$ are the given external forces.

This system is coupled with the so-called Brinkman-Forchheimer equation, which is similar to the Stokes equation and contains some convection terms similar to that in Navier-Stokes equations. However, in contrast to Navier-Stokes equations, it is shown that the global solvability in L^2-spaces holds true for the 3-dimensional problems.

Existence results for a PDE system modeling damage, in nonsmooth domains

Riccarda Rossi

University of Brescia (Italy)

We address the existence of solutions to a rate-dependent model for (incomplete) damage. The evolution of the damage parameter is ruled by a subdifferential inclusion featuring a q-laplacian type operator. We present various global existence results, progressively weakening the smoothness requirements on the spatial domain. This is joint work with Dorothee Knees (WIAS, Berlin) and Chiara Zanini (Politecnico di Torino).

Cahn-Hilliard and Thin Film equations with nonlinear mobility as gradient flows in weighted-Wasserstein metrics

Giuseppe Savaré

University of Pavia (Italy)

We discuss a novel approach to proving existence of non-negative weak solutions for degenerate parabolic equations of fourth order, like the Cahn-Hilliard and certain thin film equations. The considered evolution equations are in the form of a gradient flow for a perturbed Dirichlet energy with respect to a new class of Wasserstein-like transport metric associated to the nonlinear mobility, which should be a concave function of the spatial density. Weak solutions are obtained as curves of maximal slope by applying the variational approach to gradient flows in such a metric space: some essential properties of the solutions - non-negativity, conservation of the total mass and dissipation of the energy - are automatically guaranteed by the construction from minimizing movements in the energy landscape.

Equilibrium configurations of nematic liquid crystals on a torus

Antonio Segatti

University of Pavia (Italy)

In this talk I will report on a joint project with Marco Veneroni (Pavia) and Michael Snarski (Montreal) regarding the study of nematic liquid crystals on surfaces. In particular, I will introduce and discuss the main features of an energy recently proposed for the modelling of thin nematic shells by Napoli & Vergori. In the case of the torus, we will discuss some qualitative behavior of the equilibrium configurations.

Coupling models of phase transitions and grain boundary motions

Ken Shirakawa

Chiba University (Japan)

In this talk, some coupling models of solid-liquid phase transitions and grain boundaries are considered. These models are derived from corresponding free energies based on the ideas as in [J. A. Warren, R. Kobayashi, A. E. Lobkovski and W. C. Carter; Acta Materialia **51**, (2003), 6035-6058]. The main results presented here will be concerned with the qualitative properties for these models, such as solvabilities and energy-dissipations e.t.c., and limiting observations to understand exact relationships among these models. The related study is based on the joint work with Prof. S. Moll and Prof J. M. Mazón, Univ. Valencia, Spain.

Optimal control of Allen-Cahn equations with singular potentials and dynamic boundary conditions

Jürgen Sprekels

Weierstrass Institute, Berlin (Germany)

In this lecture, we study optimal control problems for Allen-Cahn equations with nonlinear dynamic boundary conditions involving the Laplace-Beltrami operator. Both the nonlinearities occurring in the equations governing the evolution in the bulk and on the boundary are assumed to be singular. We discuss two different cases: (i) Logarithmic singularity, (ii) singularity of double obstacle type.

In the first (differentiable) case, we are able to establish both first-order necessary and second-order sufficient optimality conditions. The case of the double obstacle nonlinearity is treated by performing a deep quench limit process, using the results obtained for the differentiable case. The resulting first-order necessary conditions do not yield as much information as in the differentiable case, but are quite typical for problems involving double obstacles.

This is a joint work with P. Colli (Pavia, parts (i) and (ii)) and H. Farshbaf-Shaker (WIAS Berlin, part (ii)).

Multi-material structural topology optimization using phase field methods

Vanessa Styles

University of Sussex (UK)

A phase field approach for structural topology optimization which allows for topology changes and multiple materials is analyzed. First order optimality conditions are rigorously derived and several numerical results for mean compliance problems are presented.

Local versus energetic solutions in rate-independent brittle delamination

Marita Thomas

Weierstrass Institute, Berlin (Germany)

This contribution addresses several models describing the rate-independent fracture of a material compound along a prescribed interface. This unidirectional process is modeled in the framework of Generalized Standard Materials with the aid of an internal delamination parameter. In the context of the energetic formulation it has become a well-established procedure to obtain solutions of the brittle model via an adhesive-contact approximation based on tools from Gamma-convergence of rate-independent systems. This means that the non-smooth, local brittle constraint, confining displacement jumps to the null set of the delamination parameter, is approximated by a smooth, non-local surface energy term. Based on this idea we present a procedure to find local solutions for the brittle model. The behavior of local and energetic solutions is compared. This is a collaboration with T. Roubicek (Prague), C. Panagiotopoulos (Seville) and R. Rossi (Brescia).

Structural stability of nonlinear flows

Augusto Visintin

University of Trento (Italy)

After Fitzpatrick's seminal work [1] it is known that in a Banach space V any maximal monotone operator $\alpha : V \to \mathcal{P}(V')$ may be given a variational representation. This is here illustrated on some examples.

On this basis De Giorgi's Γ-convergence is then applied to the analysis of monotone inclusions. Via Fitzpatrick's theory, the compactness and the structural stability of the Cauchy problem

$$\begin{cases} \partial u/\partial t + \alpha(u) \ni h & \text{in } V', \text{ a.e. in }]0,T[, \\ u(0) = u^0 \end{cases}$$

is studied, with respect to variations of the operator α and of the datum $h \in L^2(0,T;V')$. See [2].

[1] S. Fitzpatrick: Representing monotone operators by convex functions. Workshop/Miniconference on Functional Analysis and Optimization (Canberra, 1988), 59–65, Proc. Centre Math. Anal. Austral. Nat. Univ., 20, Austral. Nat. Univ., Canberra, 1988.

[2] A. Visintin: Variational formulation and structural stability of monotone equations. Calc. Var. Partial Differential Equations, **47** (2013), 273–317.

Well-posedness and stability of a hydrodynamic system modeling vesicle and fluid interactions

Hao Wu

Fudan University, Shanghai (China)

In this talk, we will discuss a hydrodynamic system modeling the deformation of vesicle membranes in incompressible viscous fluids. The system consists of the Navier-Stokes equations coupled with a fourth order phase-field equation. In the three dimensional case, we prove the existence/uniqueness of local strong solutions for arbitrary initial data as well as global strong solutions under the large viscosity assumption. We also establish some regularity criteria in terms of the velocity for local smooth solutions. Finally, we investigate the stability of the system near local minimizers of the elastic bending energy.

Continuum models of active nematics

Julia M. Yeomans

University of Oxford (UK)

Active systems, such as the cytoskeleton and bacterial suspensions, provide their own energy and hence operate out of thermodynamic equilibrium. Continuum models describing active systems are closely related to those describing liquid crystal hydrodynamics, together with an additional active stress term. We discuss how the behaviour of the active continuum models depends on model parameters, such as the strength of the activity and the liquid crystal tumbling parameter, and we compare our results to recent experiments on cytoskeletal gels.

On the cubic instability in the Q-tensor theory of liquid crystals

Arghir Zarnescu

University of Sussex (UK)

One of the most physically meaningful energy functionals expressed in terms of Q-tensors contains several terms that involve derivatives. One of these terms is a cubic term, namely:

$$\sum_{i,j,k=1}^{3} Q_{kl} \frac{\partial Q_{ij}}{\partial x_k} \frac{\partial Q_{ij}}{\partial x_l}$$

The presence of this term is known to have a "destabilising effect" in the sense that the energy functional is not bounded from below. This can be overcome for instance by introducing in the energy functional a stabilising bulk singular potential such as the one proposed by Ball and Majumdar.

We study the energy functional with the cubic term and a standard bulk potential and consider a gradient flow of the mentioned energy functional. We show that to a certain extent a dynamical theory can overcome the deficiencies of the statical theory by having global existence for small and physical initial data and blow-up for certain large enough initial data.

This is joint work with Gautam Iyer and Xiang Xu (Carnegie Mellon).

Infinite energy solutions for Cahn-Hilliard type equations in unbounded domains

Sergey Zelik

University of Surrey (UK)

The viscous Cahn-Hilliard equation in the 3D cylindrical domains in uniformly local phase spaces will be considered. In particular, the global well-posedness and dissipativity of such solutions will be established for the case of regular potentials of arbitrary polynomial growth as well as for the case of sufficiently strong singular potentials. For the cases where we have failed to prove the unique ness (e.g., for the logarithmic potentials), the existence of the trajectory attractor will be verified. Moreover, the analogous results for the so-called Cahn-Hilliard-Oono equation in the whole space $\mathbb{R}^3$ will be also discussed.

Moving martensitic phase boundaries: from micro to macro

Johannes Zimmer

University of Bath (UK)

The equations of elasticity in one space dimension, $u_{tt} = \sigma(u_x)_x$, become ill-posed if the potential energy density is nonconvex, or equivalently if σ is non-monotone. This complication necessarily arises in the theory of so-called martensitic phase transitions, which are diffusionless solid-solid transformations where several stable phases can coexist.

We want to consider the type-changing system (elliptic-hyperbolic) equation given above. In addition to selection conditions for the hyperbolic regions, we then need selection criteria for any phase-changing region, i.e., the domains where the equation becomes elliptic. These regions correspond to moving martensitic interfaces. Thus, one wants to identify selection criteria which impose conditions on moving interfaces such that the solution of a given initial-value problem becomes unique. In mechanical terms, such criteria relate the velocity of a moving interface to a driving force, and are often called kinetic relations. Phenomenological kinetic relations have been proposed, but a natural question is whether they can — at least in simple situations — be derived from first principles, namely atomistic models based on Newton's equation of motion.

To investigate this question, we first study a one-dimensional Hamiltonian chain model of martensitic materials, where neighbouring atoms are coupled by a nonlinear spring characterised by a nonconvex potential. We present some existence results for the associated nonlocal travelling wave equation and discuss non-uniqueness of microscopic solutions. The final part of the talk concerns then the macroscopic implications and discusses in particular the ensuing kinetic relation.

[1] H. Schwetlick, J. Zimmer, Arch. Rat. Mech. Anal., **206** (2012), 707-724.

[2] M. Herrmann, H. Schwetlick, J. Zimmer, Cont. Mech. Thermodyn., **24** (2012), 21-36.

[3] H. Schwetlick, J. Zimmer, SIAM J. Math. Anal., **41** (2009), 1231-1271.

POSTERS

On a Ginzburg Landau-type equation

GIACOMO CANEVARI

Laboratoire Jacques Louis Lions – UPMC, Paris (France)

In contrast with the classical Ginzburg-Landau model, in some equations arising from the theory of continuous media the order parameter space cannot be recovered as a cone over the vacuum manifold. This is the case, for instance, of the Landau-de Gennes model for liquid crystals. However, it is possible to reformulate the theory in a more general setting, in order to recover some classical results – such as clearing out, concentration of energy, and convergence properties for the minimizers – also when the cone structure is lost. We present the necessary adaptation, for an equation on a $2D-$domain.

Diffuse interface models and their sharp interface limits

JUHANNES DAUBE

University of Freiburg (Germany)

We present two diffuse interface models for liquid-vapor flow. In particular, we are interested in the corresponding sharp interface limits and interface conditions. The first model is a phase field model for quasi-incompressible flows due to Aki, Dreyer, Giesselmann, and Kraus [2]. The sharp interface models for different scalings were derived in [2,1] by means of matched asymptotic expansions. The second model known as the Navier-Stokes-Korteweg model is an extension of the compressible Navier-Stokes equations. Its static version was studied by Hermsdörfer, Kraus, and Kröner [3] and the corresponding interface conditions were obtained. In the future, we aim to derive the sharp interface model for the dynamic case of the Navier-Stokes-Korteweg equations. This is joint work with Christiane Kraus (Berlin), and Dietmar Kröner (Freiburg).

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[2] Aki, G.; Dreyer, W.; Giesselmann, J.; Kraus, C.: A quasi-incompressible diffuse interface model with phase transition. Preprint, (2012).

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On a non-isothermal diffuse interface model for two-phase flows of fluids

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We study a non-isothermal diffuse interface model for the flow of two viscous incompressible Newtonian fluids of the same density in a bounded domain $\Omega \subset \mathbb{R}^3$. We derive the model problem following the general approach proposed by M. Frémond and find a weak solution to our model problem under suitable assumptions on the data.

Asymptotic behavior of a generalized Cahn-Hilliard equation in inpainting image

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We are interested in the study of the asymptotic behavior, in terms of finite-dimentional attractors, of a generalization of the Cahn-Hilliard equation with a fidelity term (integrated over $\Omega \backslash D$ instead of the entire domain Ω). Such a model has, in particular, applications in inpainting image. The difficulty here is that we no longer have the conservation of mass, i.e. of the spatial average of u, as in the Cahn-Hilliard equation. Instead, we prove that the spatial average of the order is dissipative.

Cahn-Hilliard-Navier-Stokes systems with nonlocal interactions

SERGIO FRIGERI

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We resume the basic mathematical results concerning systems where the incompressible Navier-Stokes equations are coupled with a nonlocal Cahn-Hilliard equation. Well-posedness and asymptotic behavior will be considered with different assumptions on the double well potential free energy density and on the mobility. Particular attention will be paid on the more recent results concerning regularity, degenerate mobility and uniqueness of weak solutions.

Degenerating Cahn-Hilliard systems coupled with complete damage processes

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Complete damage in elastic solids appears when the material looses all its integrity due to high exposure. In the case of alloys, the situation is quite involved since spinodal decomposition and coarsening also occur at sufficiently low temperatures which may lead locally to high stress peaks. Experimental observations on solder alloys reveal void and crack growth especially at phase boundaries. This poster displays a degenerating PDE system defined on a time-dependent domain for phase separation and complete damage processes under time-varying Dirichlet boundary conditions. The evolution of the system is described by a degenerating Cahn-Hilliard equation for the concentration, a doubly nonlinear differential inclusion for the damage process and a degenerating quasi-static balance equation for the displacement field. All these equations are nonlinearly coupled. We introduce a suitable notion of weak solutions consisting of weak formulations of the diffusion and the momentum b alance equation, a variational inequality for the damage process and an energy inequality. For the introduced degenerating system, we prove existence of weak solutions in an SBV-framework. This is a joint work with Christiane Kraus (WIAS).

Initial-boundary value problem for the fully coupled Navier-Stokes/Q-tensor model

YUNING LIU

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In this work, we show the existence and uniqueness of local strong solution for the fully coupled Navier-Stokes/Q-tensor system on a bounded domain with Dirichlet boundary condition. One of the novelties brought in with respect to the existing literature consists in the fact that our result is valid for both 2d and 3d and for large initial data, although the existence of solution is local in time. Concerning the methodology, we use a viscosity vanishing method to handle the linearized system and the existence of solution to the nonlinear system is proved via a Banach's fixed point argument, based on the estimates on the lower order terms.

Finite volume method for the Cahn-Hilliard equation

FLORE NABET

LATP, Aix-Marseille University (France)

In this work, we propose a finite volume method for the Cahn-Hilliard equation with different boundary conditions for the order parameter. First we will study the usual Neumann boundary condition. Then, we will focus on the dynamic boundary condition which is relevant when the effective interactions between the wall and both mixture components are short-ranged. Usually,

the Cahn-Hilliard equation is discretized by finite difference, finite element or spectral methods. Here, we propose to study a 2D finite volume method which is well adapted to the coupling between the dynamics in the domain and the one on the boundary. We show some theoretical results and provide numerical illustrations for the two boundary conditions.

Image segmentation using a multi-phase field model

Gary Poole

University of Sussex (UK)

We use a modified vector valued Allen-Cahn equation together with the primal dual active set method to segment colour images.

Phase transitions and precipitate formation in geothermal reservoirs

Georgy Tsypkin

Institute for Problems in Mechanics, Russian Academy of Sciences, Moscow (Russia)

Processes in porous permeable rock in which solid material is deposited from solution or is removed due to dissolution are very important for different applications. A mathematical model of a fresh water injection into geothermal reservoir saturated with superheated vapor and solid salt is presented. Fresh water dissolves solid salt near injection well-bore and transfers it towards the extraction well. Then water evaporates in the domain of low pressure where salt precipitates again. Analysis of derived similarity solution shows that the solution is two-valued and ceases to exist at critical parameters which correspond to the reservoir sealing with solid phase of salt.

Entropy solutions to the zero-flux problem for strongly degenerate parabolic equations

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Strongly degenerate parabolic equations are regarded as a linear combination of the time-dependent conservation laws (quasilinear hyperbolic equations) and the porous medium type equations (nonlinear degenerate parabolic equations). Thus, this equation has both properties of hyperbolic equation and those of parabolic equations and describes various nonlinear convective diffusion phenomena such as filtration problems, Stefan problems and so on. In this talk we consider the zero-flux problem for strongly degenerate parabolic equations. In particular, we show the existence and uniqueness of entropy solutions to such a problem.
