3.36pt

Classification

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Supervised Learning

Definition 1 (Supervised Learning)

Supervised Learning is the task of learning (inferring) a function f that maps input vectors to their corresponding target vectors, by using a dataset containing a given set of pairs of $(input, output)$ samples. Examples:

- Regression: the output vectors take one or more continuous values.
- Classification: the output vectors take one value of a finite number of discrete categories. Special case: binary classification.

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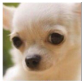
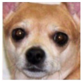
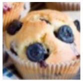
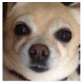
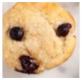
$\boldsymbol{x}_1 =$  $\boldsymbol{x}_2 =$  $\boldsymbol{x}_3 =$  $\boldsymbol{x}_4 =$  $\boldsymbol{x}_5 =$ 

$y_1 = 1$ $y_2 = 1$ $y_3 = -1$ $y_4 = 1$ $y_5 = -1$

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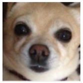
$\boldsymbol{x}_1 =$ $\boldsymbol{x}_2 =$ $\boldsymbol{x}_3 =$ $\boldsymbol{x}_4 =$  $\boldsymbol{x}_5 =$

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$$f : X \to Y \qquad \text{where } f := f_m \circ f_{m-1} \circ \ldots f_1$$

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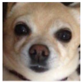
$\mathbf{x}_1 =$ $\mathbf{x}_2 =$ $\mathbf{x}_3 =$ $\mathbf{x}_4 =$  $\mathbf{x}_5 =$

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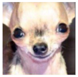
2. Use a Test Set to validate f:

$f\Big(\mathbf{x}_6 =$ $\Big) = -1$ $f\Big(\mathbf{x}_7 =$  $\Big) = 1$

Classification models

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Multi-class Learning: involves finding the boundaries that separates more than two classes from each other.

Classification (basic) Methods

1. *k*-**Nearest Neighbours**: New examples are assigned a class based on how similar, using a **METRIC (!)**, they are to examples in the training data. The *learned* model is the training examples itself.

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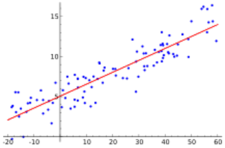

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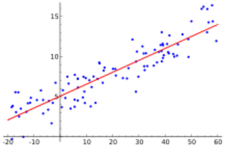

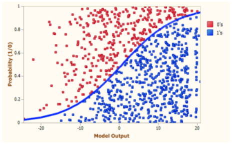

3. **Logistic Regression**: We can train a logistic regression model (see, Chap. 4.4 in Hastie et all.)

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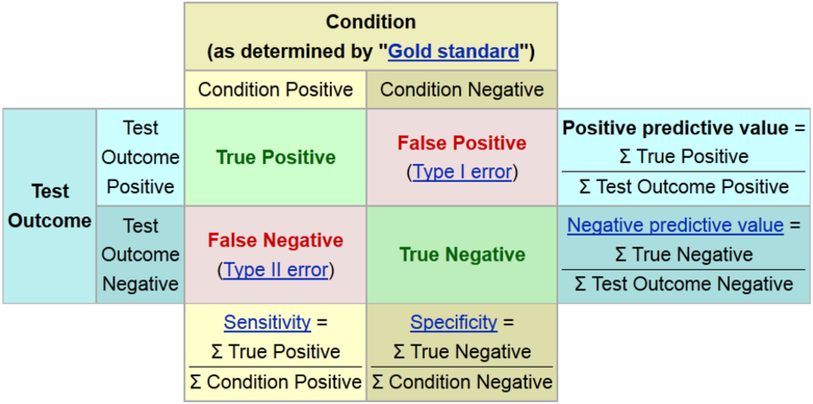
... but **BEST RESULTS** with respect to which METRIC?

Binary Classification: Confusion Matrix



		Condition (as determined by "Gold standard")		
		Condition Positive	Condition Negative	
Test Outcome	Test Outcome Positive	**True Positive**	**False Positive** (Type I error)	**Positive predictive value** = $\frac{\Sigma \text{ True Positive}}{\Sigma \text{ Test Outcome Positive}}$
	Test Outcome Negative	**False Negative** (Type II error)	**True Negative**	Negative predictive value = $\frac{\Sigma \text{ True Negative}}{\Sigma \text{ Test Outcome Negative}}$
		Sensitivity = $\frac{\Sigma \text{ True Positive}}{\Sigma \text{ Condition Positive}}$	Specificity = $\frac{\Sigma \text{ True Negative}}{\Sigma \text{ Condition Negative}}$	

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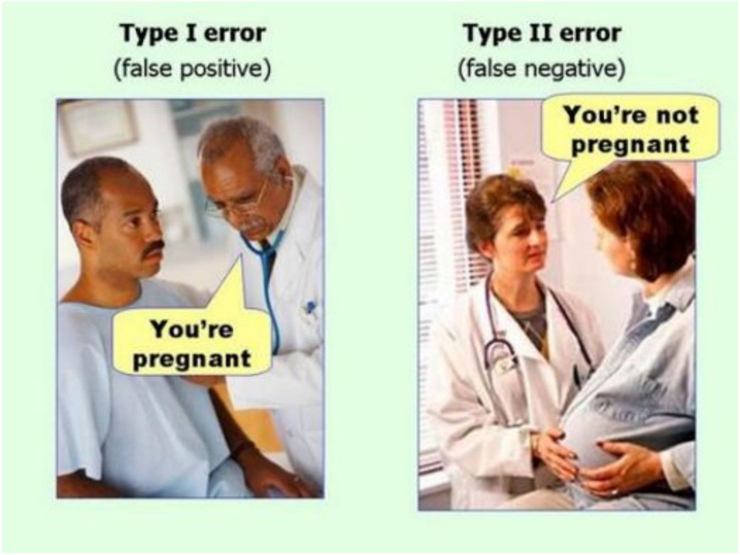

Accuracy

$$\textsc{Accuracy} := \frac{\#\mathsf{TP} + \#\mathsf{TN}}{\#\mathsf{TP} + \#\mathsf{TN} + \#\mathsf{FP} + \#\mathsf{FN}}$$

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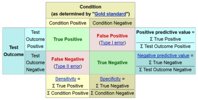
QUESTION: When Accuracy make sense for evaluating a binary classifier? Example?

Evaluating Classifiers with Imbalanced Classes



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		Sensitivity = Σ True Positive / Σ Condition Positive	Specificity = Σ True Negative / Σ Condition Negative	

$$\textsc{Sensitivity (Recall)} := \frac{\#\mathsf{TP}}{\#\mathsf{TP} + \#\mathsf{FN}}$$

$$\textsc{Specificity (Precision)} := \frac{\#\mathsf{TN}}{\#\mathsf{TN} + \#\mathsf{FP}}$$

$$\textsc{Positive Predictive Value} := \frac{\#\mathsf{TP}}{\#\mathsf{TP} + \#\mathsf{FP}}$$

$$\textsc{Negative Predictive Value} := \frac{\#\mathsf{TN}}{\#\mathsf{TN} + \#\mathsf{FN}}$$

Evaluating a Classifiers: Loss Functions

1. **Mean Square Error/Quadratic Loss/L2 Loss**:

$$MSE := \frac{1}{n}\sum_{i=1}^{n}(y_i - f(x_i; w))^2$$

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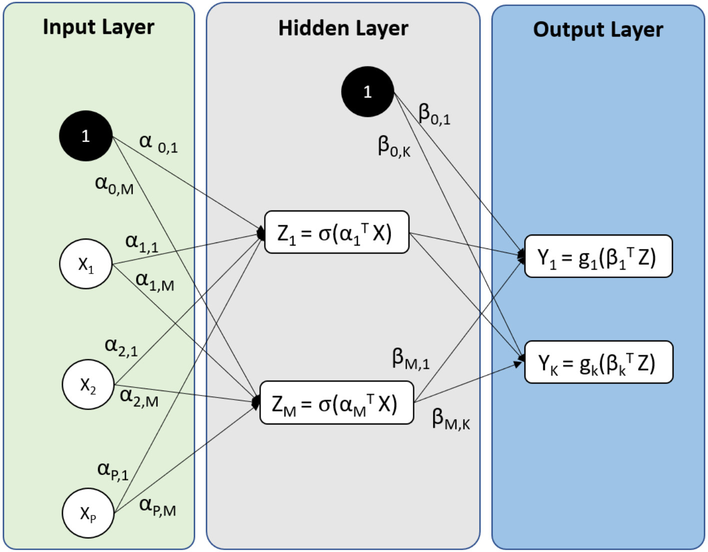
3. **Cross Entropy Loss/Negative Log Likelihood**:

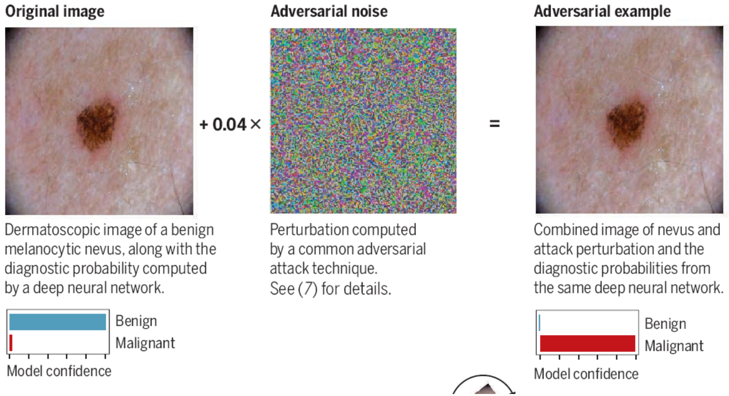
$$CrossEntropy := \sum_{i=1}^{n} -(y_i \log(f(x_i; w)) + (1 - y_i)\log(1 - f(x_i; w)))$$

Classification with Multilayer Neural Networks



$\alpha \in R^{P\times M}$ $\beta \in R^{M\times K}$

Generative Adversarial Attacks

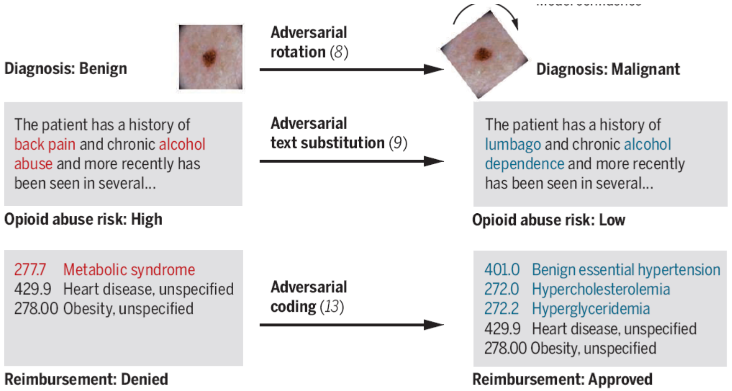


Dermatoscopic image of a benign melanocytic nevus, along with the diagnostic probability computed by a deep neural network.

Perturbation computed by a common adversarial attack technique. See (7) for details.

Combined image of nevus and attack perturbation and the diagnostic probabilities from the same deep neural network.

Generative Adversarial Attacks



Diagnosis: Benign

Adversarial rotation *(8)*

Diagnosis: Malignant

The patient has a history of back pain and chronic alcohol abuse and more recently has been seen in several...

Adversarial text substitution *(9)*

The patient has a history of lumbago and chronic alcohol dependence and more recently has been seen in several...

Opioid abuse risk: High

Opioid abuse risk: Low

277.7	Metabolic syndrome
429.9	Heart disease, unspecified
278.00	Obesity, unspecified

Adversarial coding *(13)*

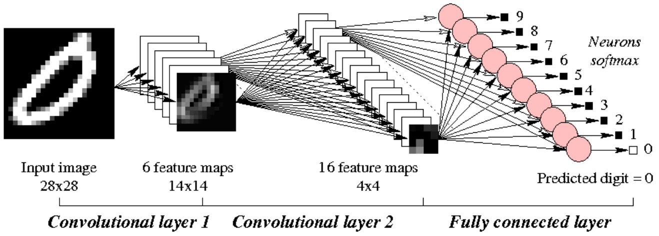
401.0	Benign essential hypertension
272.0	Hypercholesterolemia
272.2	Hyperglyceridemia
429.9	Heart disease, unspecified
278.00	Obesity, unspecified

Reimbursement: Denied

Reimbursement: Approved

Convolutional NN

(a)



(b)

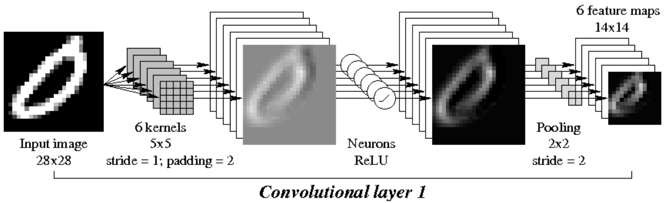


Figure 1: A convolutional neural network for the MNIST problem: global architecture (a) and detailed view of the first convolutional layer (b).

Recurrent NN

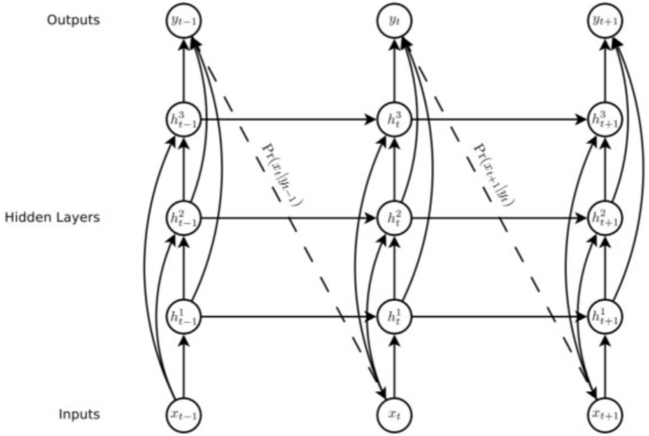


Figure 1: **Deep recurrent neural network prediction architecture.** The circles represent network layers, the solid lines represent weighted connections and the dashed lines represent predictions.

Elegant Machine Learning: Flux