Special session at **Mathematics and its applications**
a joint SIMAI-SMAI-SMF-UMI meeting. Torino, July 3-7, 2006

Modelling and analysis of phase transitions and free boundary problems

July 4, 2006

Department of Mathematics of the "Politecnico di Torino"

Tentative program

9:15 – 9:45 : Gianni Gilardi, *University of Pavia*

Some singular systems of phase field type

Recently, a system that couples a singular integrodifferential equation with an Allen-Cahn-type equation has been presented in joint work with E. Bonetti, P. Colli, and M. Fabrizio in order to provide a thermodynamic model for phase transitions with memory. In a couple of papers, the well-posedness of the mathematical problem is proved and the long time behaviour of the solution is studied.

More recently, an asymptotic analysis of the above system has been performed in a joint work with E. Rocca. This leads to a new model for transition/separation processes (with or without memory) that is described by a singular phase relaxation system.

In the last days, E. Rocca and I have studied the possibility of replacing the Allen-Cahn equation with the Cahn-Hilliard equation in previous systems.

In this talk, I present an overview of the results comig out from such colllaborations.

9:45 – 10:15 : Morgan Pierre, *University of Poitiers*

Symmetric harmonic maps describing equilibria of nematic crystals in a cylinder

The liquid crystal phase is a transitional phase between liquid and solid. In the Oseen-Franck model, the liquid crystal is represented by an S^2-valued map, and (in the model case) its energy reduces to the Dirichlet energy. The problem of finding stable configurations of the liquid crystal becomes that of finding locally minimizing S^2-valued harmonic maps. In this presentation, we shall focus on the problem of multiple solutions for the Dirichlet problem for harmonic maps from the unit disc in R^2 into S^2, in relation with the symmetries of the problem. We shall also give numerical examples.

10:15 – 10:45 : **Coffee break**

10:45 – 11:15 : Pierluigi Colli, *University of Pavia*

Large time behaviour of some singular phase transition models

This talk is concerned with the large time behaviour of the solution to some phase transition problems of phase field type. The initial-boundary value problem for the related system of partial differential equations is considered and the convergence properties of the trajectories as time goes to infinity are discussed.

11:15 – 11:45 : Laurent Chupin, *INSA of Lyon*

Numerical and theoretical results for a mixture of viscoelastic flows using the Cahn-Hilliard formulation

We are first interested in the derivation of a model for the study of the viscoelastic diphasic flows. Beyond the Navier-Stokes equations describing the hydrodynamic properties of a fluid, we express on the one hand the diphasic character while using an order parameter (volumic fraction of one phase in the mixture) and by taking of account the theories of Cahn and Hilliard on the thermodynamic interactions between the two phases, on the other hand the viscoelastic effect thanks to a behavioral law of Oldroyd type.

In this presentation, we study the mathematical properties of the model (existence and unicity results) and then we validate qualitatively this model *via* a numerical scheme. Several applications close to physical contexts precise good are carried out.

11:45 – 12:15 : Ugo Gianazza, *University of Pavia*

Intrinsic Harnack estimates for degenerate parabolic differential equations

We establish the intrinsic Harnack inequality for non-negative solutions of a class of degenerate, quasilinear, parabolic equations, including equations of the p-Laplacian and porous medium type. It is shown that the classical Harnack estimate continues to hold, when considered in a space-time geometry intrinsic to the degeneracy. Hölder estimates are then derived as a consequence of the Harnack inequality. This talk reports on a joint work with E. DiBenedetto and V. Vespri.

12:15 – 12:45 : Daniele Andreucci, *University of Rome "La Sapienza"*

Nonlinear convection terms in degenerate parabolic equations.

We consider second order degenerate parabolic equations containing nonlinear terms depending on the first order spatial gradient of the solution.

We investigate some features of solutions like: finite speed of propagation, asymptotic stabilization for large times, integral and maximum estimates, in dependence of the relative size of the various nonlinearities entering the structure of the equation.

12:45 – 15:15 : **Lunch time**

15:15 – 15:45 : Alain Miranville, *University of Poitiers*

Nonisothermal Allen-Cahn models

We are interested in this talk in the study of nonisothermal Allen-Cahn equations. Such models are obtained based on a microforce balance introduced by M. Gurtin; we can note that they bear some resemblance to the models derived by a similar approach due to M. Frémond and based on a balance for microscopic movements.

15:45 – 16:15 : Maurizio Grasselli, *Polytechnic of Milan*

Parabolic-hyperbolic phase-field systems

We intend to present some results about the longterm behavior of solutions to initial and boundary value problems for a nonlinear parabolic-hyperbolic system of partial differential equations. This system, connected with the description of phase transition phenomena, consists of a parabolic equation governing the (relative) temperature which is coupled with a damped semilinear wave equation ruling the evolution of the order parameter (or phase-field). With respect to the standard parabolic phase-field system proposed by G. Caginalp, the kinetic equation for the order parameter has an additional term, that is, a second order time derivative multiplied by a (small) inertial parameter. Existence of global as well as exponential attractors and the convergence to equilibria will be the main topics of this talk.

16:15 – 16:45 : **Coffee break**

16:45 – 17:15 : Alberto Farina, *University of Amiens*

Phase transitions, superconductors and superfluids

In this talk I intend to present some results concerning finite-energy solutions of the Ginzburg-Landau equation modelling superconductors and the Gross-Pitaevskii equation arising in the study of superfluids.

17:15 – 17:45 : Augusto Visintin, *University of Trento*

Homogenization of a nonlinear PDE via two-scale convergence

Some properties of two-scale convergence (in the sense of Nguetseng) are first outlined. The remainder of this talk will illustrate how this notion can be applied to a nonlinear problem at the PDEs.

Maxwell's equations in a nonhomogeneous and anisotropic metal surrounded by an insulating enviroment lead to a parabolic-hyperbolic system of partial differential equations in the whole space. We provide a weak formulation for an initial-value problem with nonlinear constitutive relations of the form

$$\vec{B} = \vec{\mathcal{F}}(\vec{H}, x), \qquad \vec{J} = \vec{\mathcal{G}}(\vec{E}, \vec{H}, x).$$

We allow for possible discontinuities of $\mathcal{F}(\cdot, x)$, thus accounting for the possible occurrence of free boundaries, but we neglect hysteresis.

We prove the existence of a solution. We then assume that the above constitutive relations exhibit fast periodic oscillations in space, let the space-period vanish, and prove two-scale convergence (in the sense of Nguetseng) to a two-scale homogenized problem. We also derive a single-scale homogenized problem.

A simpler analogous problem is issued from phase transitions. Open questions include the possible uniqueness of the solution.

17:45 – 18:15 : Michel Frémond, *Laboratoire Lagrange, Paris*

Volume and surface damage. A predictive theory

This talk reports on a joint work with Elena Bonetti (Pavia) and Francesco Freddi (Parma). A model coupling damage of domains and damage of interfaces is described. Because damage results from microscopic motions, the power of these microscopic motions is included in the power of the internal forces. The power of the internal forces depends on the damage velocity and on its gradient to take into account local interactions. An interaction between the domain damage and the damage along the interface is introduced. In order to overcome the insensitiveness of the local interface model to elongation, non-local elongation has been considered as a source of damage. Numerical examples confirm the effectiveness of the proposed model to describe various damage phenomena in agreement with experiments, for instance for two concrete elements glued on one another.

The model involves coupled partial differential equations both in the domains and the interfaces. Non local quantities due to the terms accounting for the elongation appear in the interface partial differential equation. An existence theorem may be proved provided some assumptions are satisfied.
