Master Program in Electronic Engineering

Advanced Mathematical Methods for Engineers

February 20, 2017

1. Consider the Cauchy Problem

$$\begin{cases} y'(x) = -\dfrac{y(x)}{1+e^{y(x)}} \\ y(0) = 1 \end{cases}$$

Study the problem of local and global existence of solutions (hint: compute $f_y(x,y)$, where $f(x,y) = -\frac{y(x)}{1+e^{y(x)}}$), and draw the qualitative graph, establishing in particular if there are asymptots for the graph.

2. Given the ODE system

$$\begin{cases} x' = 3x - 2y \\ y' = 2x - 2y \end{cases}$$

find:

a) all solutions on $[0, +\infty)$;

b) the bounded solutions on $[0, +\infty)$.

3. Given the sequence of functions for $n \geq 1$ and $x \in E = [4, 6]$:

$$f_n(x) = \frac{x^3 + 3(\sqrt{n} - 2)x^2 - 24\sqrt{n}x - 2}{n^2}$$

a) verify that $f_n \in L^1(E)$ for every $n \geq 1$;

b) find f such that $f_n \to f$ pointwise in E;

c) verify that $f_n \to f$ in $L^1(4, 6)$;

d) compute the $\lim_{n\to+\infty} \int_4^6 f_n(x)\, dx$

4. Given the sequences of functions $g_n(x) = 1 - |x - n - 1|$ and $f_n(x) = g_n^+(x) := \max\{0, g_n(x)\}$. Find

a) the pointwise limit of f_n on $\mathbb{R}$;

b) the limit in ($C^0([a, b]; \mathbb{R})$ with the sup-norm) of f_n.

making explicit the computations.