Master Program in Electronic Engineering

Advanced Mathematical Methods for Engineers

June 21, 2017

1. Consider the Cauchy Problem

$$\begin{cases} y' = (y-2)^2 \sin x \\ y(0) = a \end{cases}$$

Find the explicit expression of the solutions as a ranges in $\mathbf{R}$. Moreover, determine for which values of $a \in \mathbf{R}$ the solution has domain the whole $\mathbf{R}$.

2. Given the ODE system

$$\begin{cases} x' = y \\ y' = -x - y - (x^2 + y^2) \end{cases}$$

a) find the critical points;

b) study their stability writing down the corresponding linearized system.

3. Given the sequence of functions for $n \geq 1$ and $x \in [0, +\infty)$:

$$f_n(x) = \frac{2x^3}{\pi(n + x^4)} \arctan\left(\frac{1}{nx^2 + 2}\right)$$

a) find f such that $f_n \to f$ pointwise in $[0, +\infty)$ as $n \to +\infty$;

b verify that $f_n \to f$ in $C^0([0, +\infty))$ with the sup-norm $\|g\| = \sup_{x \in [0,+\infty)} |g(x)|$.

4. Using the method of separation of variables, determine the solution of the following Initial-Boundary Value Problem

$$\begin{cases} u_t(x,t) - u_{xx}(x,t) = 0 & \text{for } (x,t) \in (0,\pi) \times \mathbb{R}^+ \\ u(x,0) = f(x) & \text{for } x \in (0,\pi) \\ -u_x(0,t) = 0, \quad u_x(\pi,t) = 0 & \text{for } t \in \mathbb{R}^+ \end{cases}$$

where $f \in C^1([0,\pi])$, $f'(0) = f'(\pi) = 0$. Discuss then the uniqueness of solutions by using the energy inequality.