Master Program in Electronic Engineering

Advanced Mathematical Methods for Engineers

March 18, 2019 (Appello Straordinario)

1. Consider the following Cauchy Problem

$$\begin{cases} y'(x) = |y(x)|(1 - y(x))\dfrac{x^3}{1+x^4} \\ y(0) = 2 \end{cases}$$

1.1) Discuss local and global existence and uniqueness of solutions.

1.2) Find the explicit solution (with the respective domain) and draw a qualitative graph.

2. Discuss for $\lambda \neq 0$ the existence and uniqueness of solutions for the boundary value problem:

$$\begin{cases} y'' + 2\lambda y' + 2\lambda^2 y = 2\left(x + \frac{1}{\lambda}\right), \\ y(0) = 0 \\ y(\pi) = \frac{\pi}{4} \end{cases}$$

and find explicitely the solutions when they exist.

3. Compute (rigorously justifying the passages) the limit:

$$\lim_{n\to+\infty} \int_n^{n+2} \frac{2}{(n+2)^3} \left[(x-n)(x-n-2)\right]\, dx.$$

4. Determine all solutions u in $\mathcal{D}'(\mathbf{R})$ of the equation

$$(x^3 - 8)u' = \delta_0',$$

where δ_0' denotes the derivative in $\mathcal{D}'(\mathbf{R})$ of the Dirac delta δ_0.