Global attractor for gradient flows of non-convex functionals and applications

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Abstract

In this talk we present some results concerning the long-time behavior of the solutions of the following gradient flow equation in the Hilbert space $\mathcal{H}$

$$\begin{cases} u'(t) + \partial_s \varphi u(t) \ni f & \text{for a.e. } t \in (0\,, T) \\ u(0) = u_0, \end{cases}$$

where $\varphi : \mathcal{H} \to (-\infty, +\infty]$ is a proper and lower semicontinuous functional which is not supposed to be a smooth perturbation of a convex functional and $\partial_s \varphi$ is a suitable limiting version of its subdifferential. The lack of convexity prevents us to prove a uniqueness result for the solutions of this equation. The existence of a global attractor, which attracts all the trajectories of the system with respect to a metric strictly linked to the constraint imposed on the unknowns, is thus obtained by making use of the theory of Generalized Semiflows recently proposed by J.M. Ball. As a byproduct of this abstract theory, we construct a global attractor for the so-called *quasistationary phase field model*.
This is a joint work with R. Rossi (Brescia) and U. Stefanelli (Pavia).
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