Stability properties for quasilinear parabolic equations with measure data and applications

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Abstract

Let $\Omega$ be a bounded domain of $\mathbb{R}^N$, and $Q = \Omega \times (0, T)$. We first study problems of the model type

$$
\begin{cases}
    u_t - \Delta_p u = \mu & \text{in } Q, \\
    u = 0 & \text{on } \partial \Omega \times (0, T), \\
    u(0) = u_0 & \text{in } \Omega,
\end{cases}
$$

where $p > 1$, $\mu \in \mathcal{M}_b(Q)$ and $u_0 \in L^1(\Omega)$. Our main result is a stability theorem extending the results of Dal Maso, Murat, Orsina, Prignet, for the elliptic case, valid for quasilinear operators $u \mapsto -\text{div}(A(x, t, \nabla u))$.

As an application, we consider perturbed problems of type

$$
\begin{cases}
    u_t - \Delta_p u + \mathcal{G}(u) = \mu & \text{in } Q, \\
    u = 0 & \text{on } \partial \Omega \times (0, T), \\
    u(0) = u_0 & \text{in } \Omega,
\end{cases}
$$

where $\mathcal{G}(u)$ may be an absorption or a source term. In the model case $\mathcal{G}(u) = \pm |u|^{q-1}u$ ($q > p - 1$), or $\mathcal{G}$ has an exponential type. We give existence results when $q$ is subcritical, or when the measure $\mu$ is good in time and satisfies suitable capacity conditions.