Locally Refined B-splines for isogeometric representation and Isogeometric Analysis

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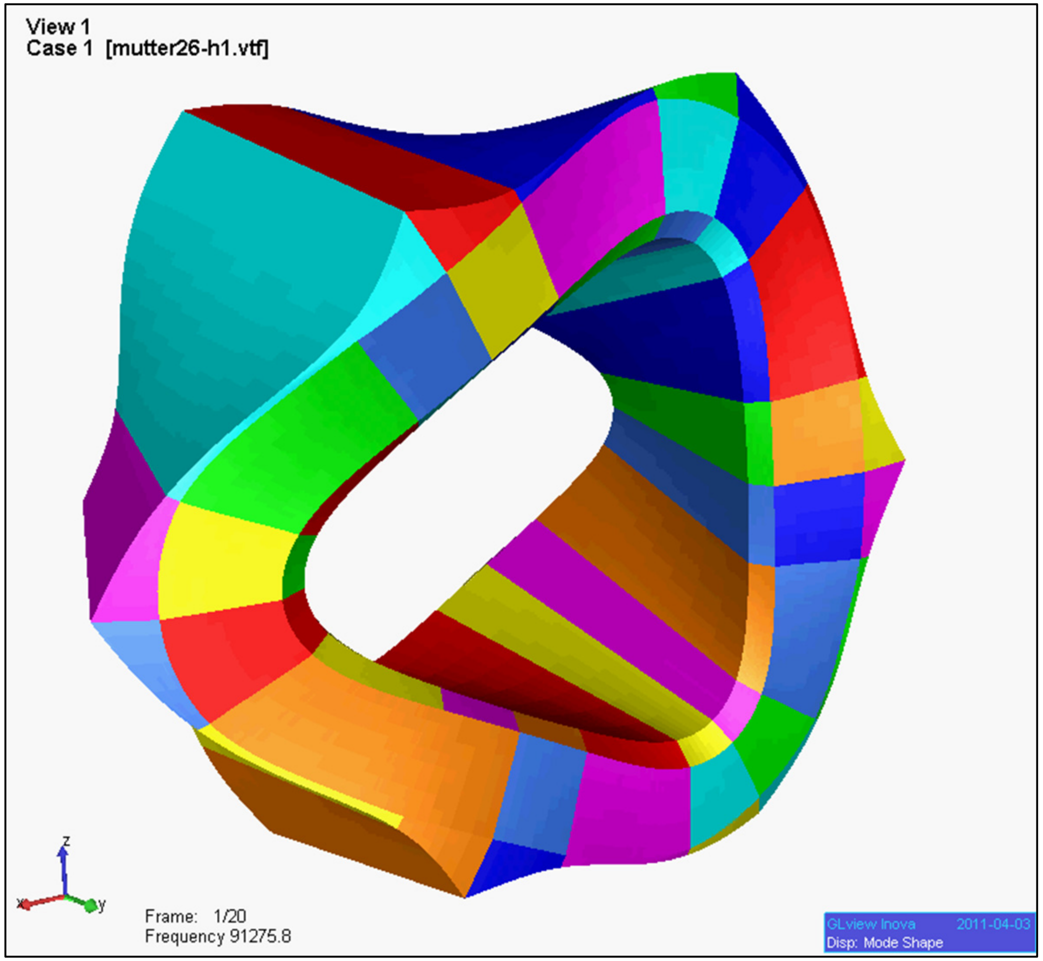
Free vibration of a Nut
– 3-variate NURBS elements



Example by: Knut Morten Okstad, SINTEF IKT

Introductory remarks

The idea of LR B-splines is inspired by T-splines

- The work on LR B-splines started in June 2009 when I wanted to understand T-splines better. However, I did this without looking at the details of the T-spline approach, contemplating the concept "Local Refinement of B-spline represented functions".
- Comparing my thoughts on "Local Refinement of B-spline represented functions" with T-splines, we realized we had an alternative approach. The LR B-spline refinement is specified in the parameter domain, while for T-splines the B-splines are inferred from T-splines vertex grid,
- The LR B-splines can be given a T-spline type interface for specifying refinement. Having such an interface the data provided will be used directly to refine in the parameter domain, rather than reason in the T-spline type vertex grid.

Introductory remarks - continued

- We prefer to refine directly in the mesh in the parameter domain, as we then deal with only one mesh that reflects the piecewise polynomial structure, rather than the multiple meshes of T-splines.
- It was also obvious that the idea of local refinement posed many new questions related to what we now denote B-splines over Box-partitions. So rather than first proposing new algorithms we: Prof Tom Lyche University of Oslo, Tor Dokken and Kjell Fredrik Pettersen from SINTEF wanted to build a theoretical foundation.
- The theoretical work took much longer time than originally anticipated. The preprint (March 2012) of the paper can be downloaded from:
  - http://www.sintef.no/upload/IKT/9011/geometri/LR-splines%20SINTEF%20Preprint%20-%20signatures.pdf

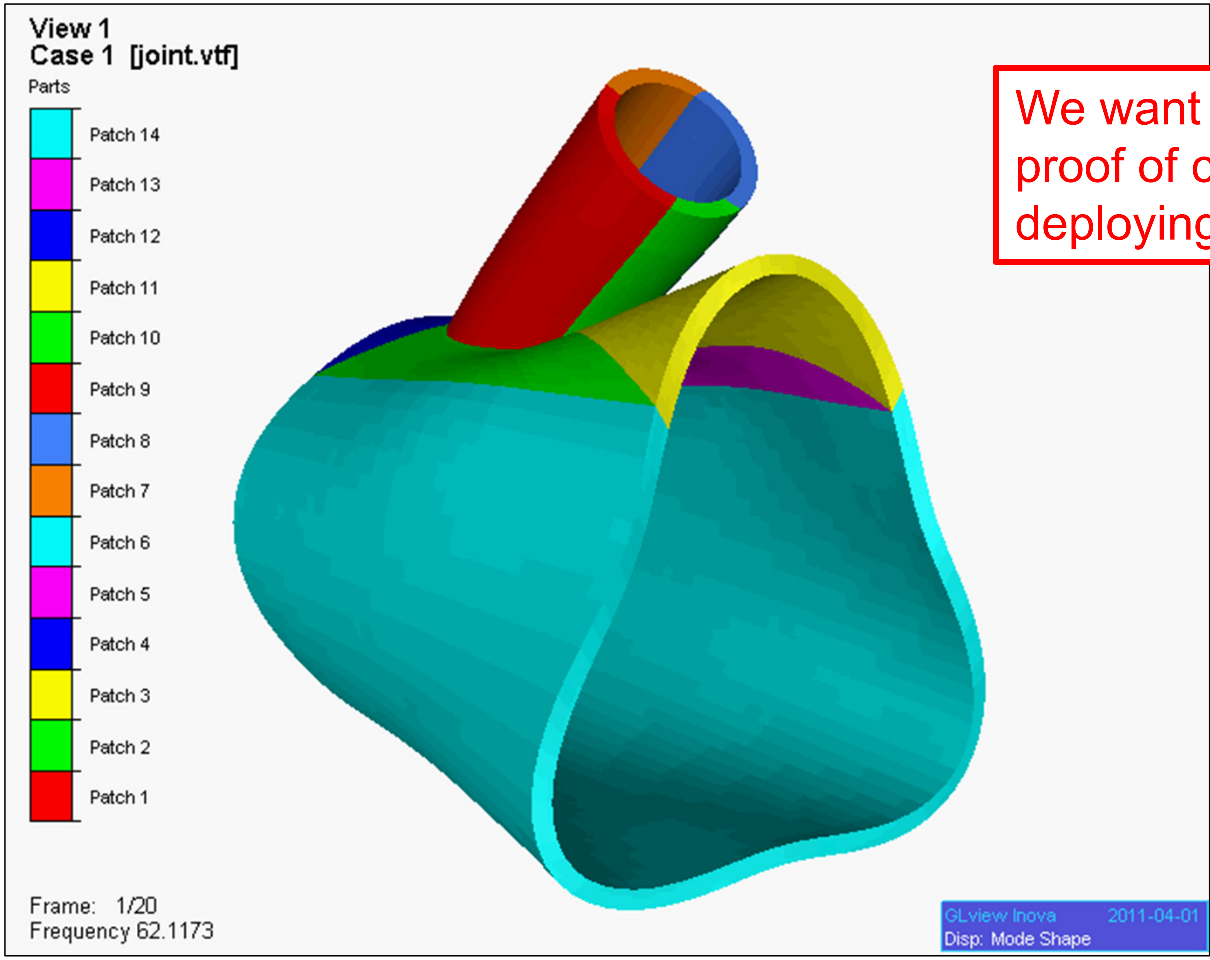
Structure of the 4 lectures

- Hour 1 givens and introduction and sets the scene.
- Hour 2 will be focused on spline spaces over box-partitions and their relevance for FEA. The lectures are thus relaxant for both:
  - T-splines
  - PHT-splines
  - LR B-splines
  - Hierarchical splines
- Hour 3 and 4 will address LR B-splines

Introduction - The larger picture

Some of you will have seen some/many of these slides before, but I feel it important to paint a larger picture.

Free vibration of a Tubular Joint – 3-variate NURBS elements



We want to move from proof of concept of IGA to deploying IGA in industry.

Example by: Knut Morten Okstad, SINTEF ICT

Independent evolution of Computer Aided Design (CAD) and Finite Element Analysis (FEA)

- CAD and FEA evolved in different communities before electronic data exchange
  - FEA developed to improve analysis in engineering
  - CAD developed to improve the design process
  - Information exchange was drawing based, consequently the mathematical representation used posed no problems
    - Manual modelling of the element grid
  - Implementations used approaches that best exploited the limited computational resources and memory available.
- FEA was developed before the NURBS theory
  - FEA evolution started in the 1940s and was given a rigorous mathematical foundation around 1970 (1973: Strang and Fix's *An Analysis of The Finite Element Method)*
  - B-splines: 1972: DeBoor-Cox Algorithm, 1980: Oslo Algorithm

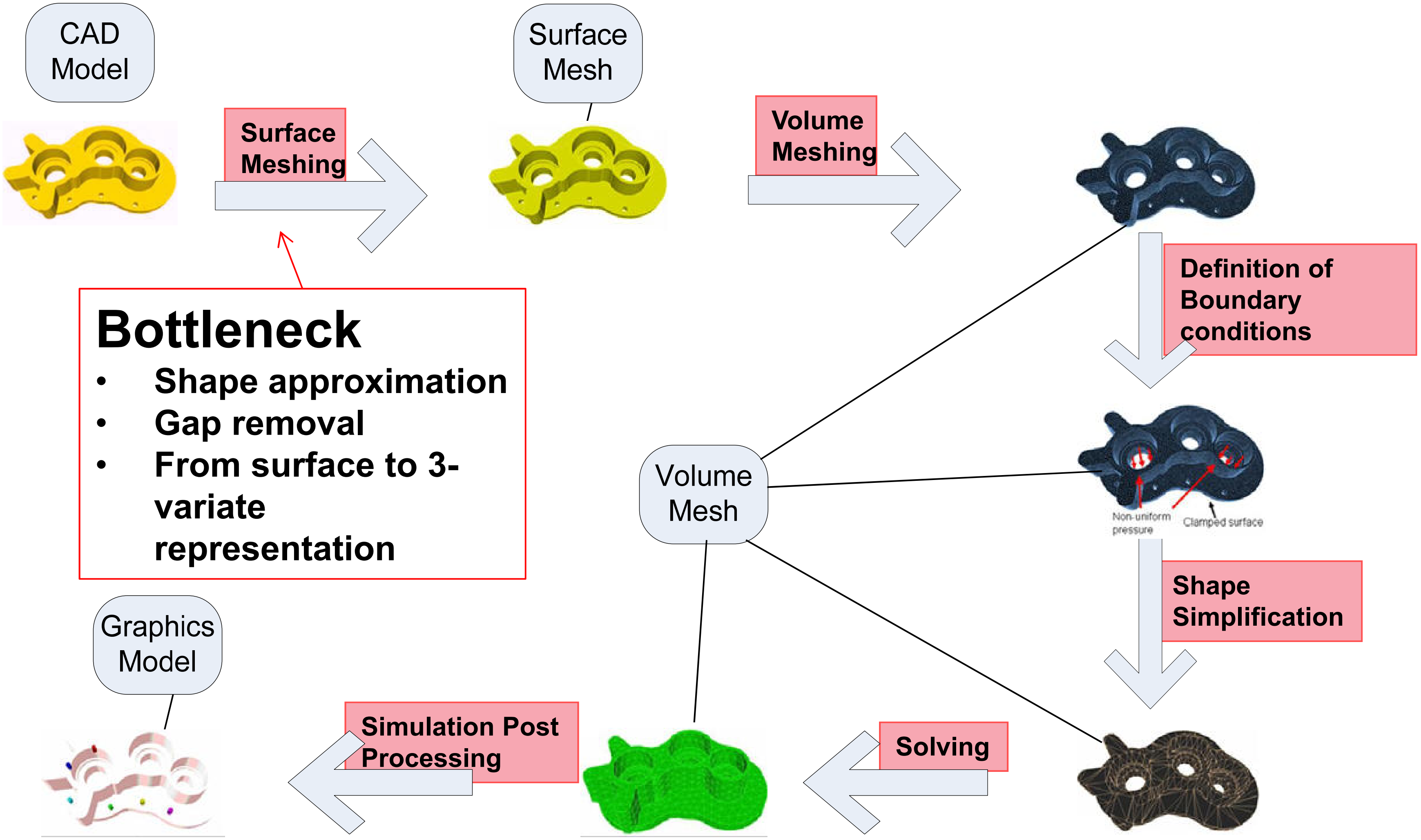
Why have NURBS not been used in FEA?

- Shape representation in CAD and FEA
  - Finite Elements are 3-variate polynomials most often of degree 1 or 2 and cannot represent many shapes accurately.
  - NURBS (NonUniform Rational B-Spline) surfaces used in CAD can represent elementary curves and surfaces exactly. (circle, ellipse, cylinder, cone…) in addition to free form shapes.
  - Mathematical representation in CAD and FEA chosen based on what was computationally feasible in the early days of the technology development.
- We needed someone with high standing in FEA to promote the idea of splines in analysis
  - Prof. Tom Hughes, University of Texas at Austin, did this in 2005
  - This has triggered a new drive in spline research after a quiet period.

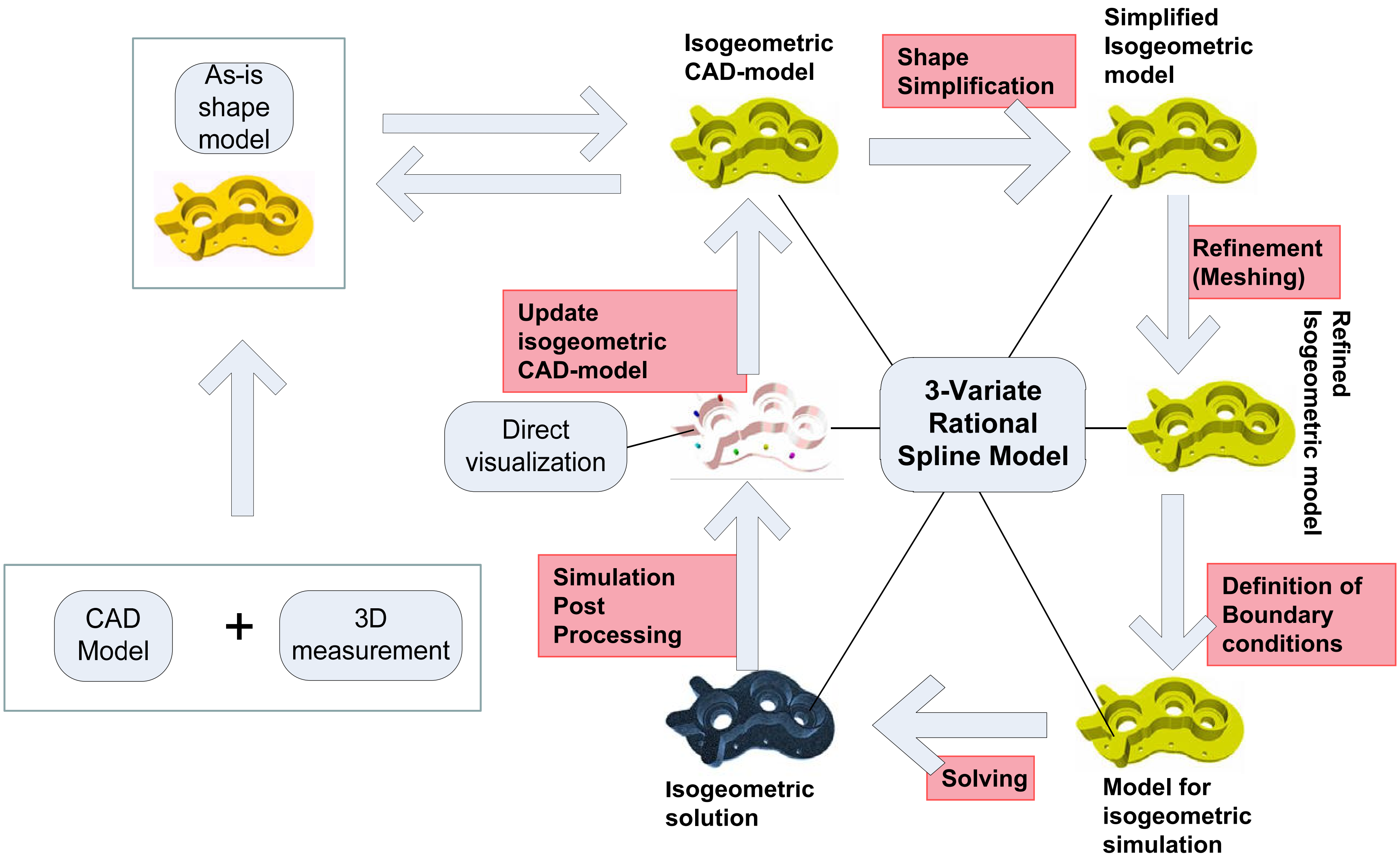
Why are splines important to isogeometric analysis?

- Splines are polynomial, same as Finite Elements
- B-Splines are very stable numerically
- B-splines represent regular piecewise polynomial structure in a more compact way than Finite Elements
- NonUniform rational B-splines can represent elementary curves and surfaces exactly. (circle, ellipse, cylinder, cone…)
- Efficient and stable methods exist for refining the piecewise polynomials represented by splines
  - Knot insertion (Boehms Algorithm, and Oslo Algorithm, 1980)
  - B-spline has a rich set of refinement methods

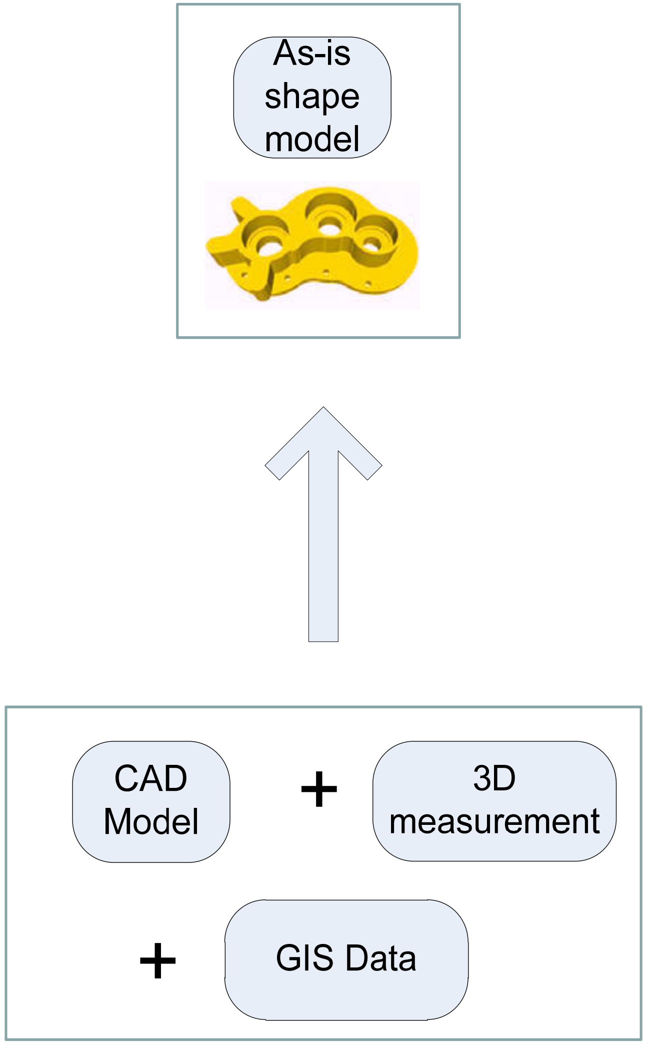
Traditional simulation pipeline



Simulation – Future Information flow

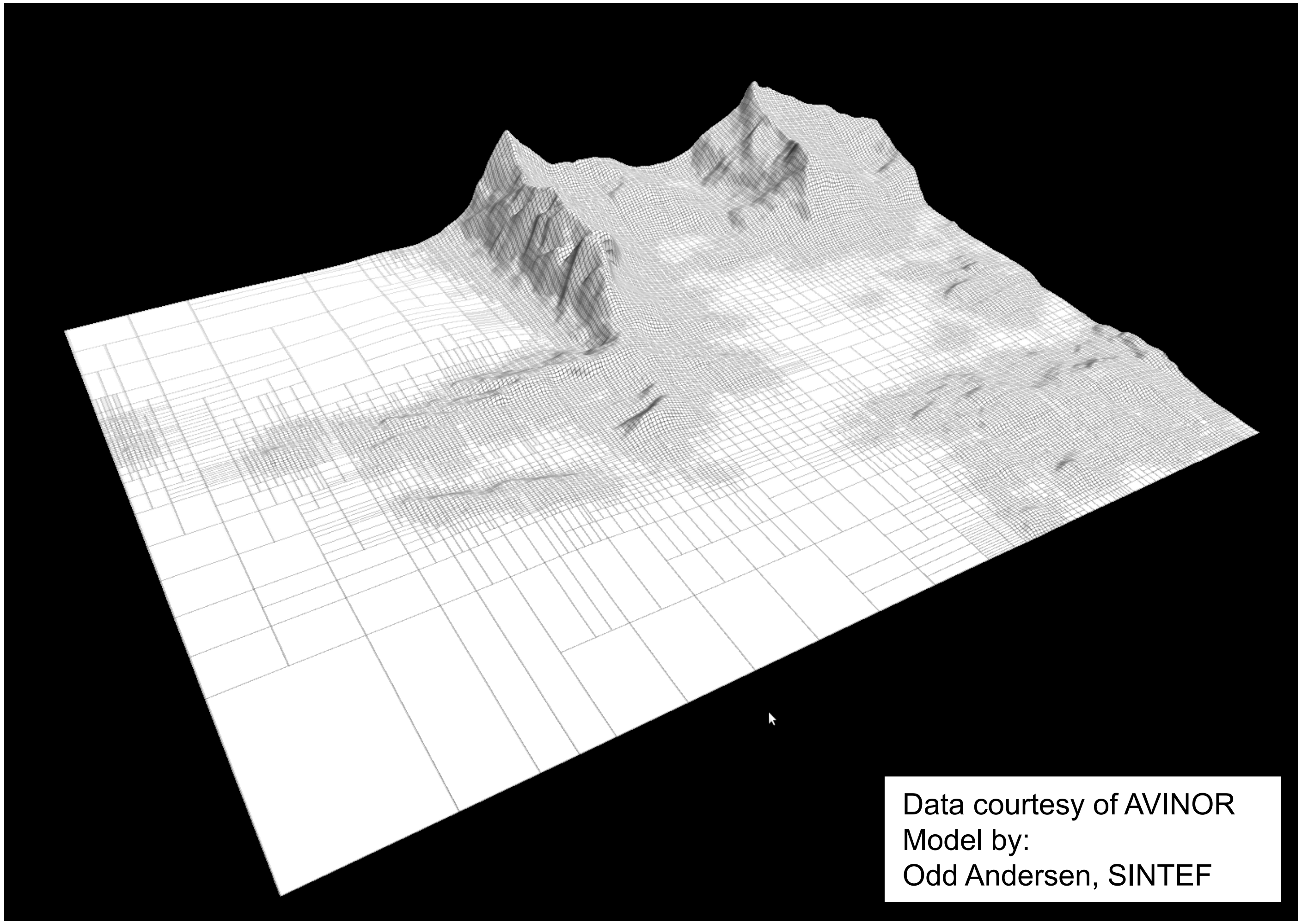


Challenge 1: Create “as-is” model



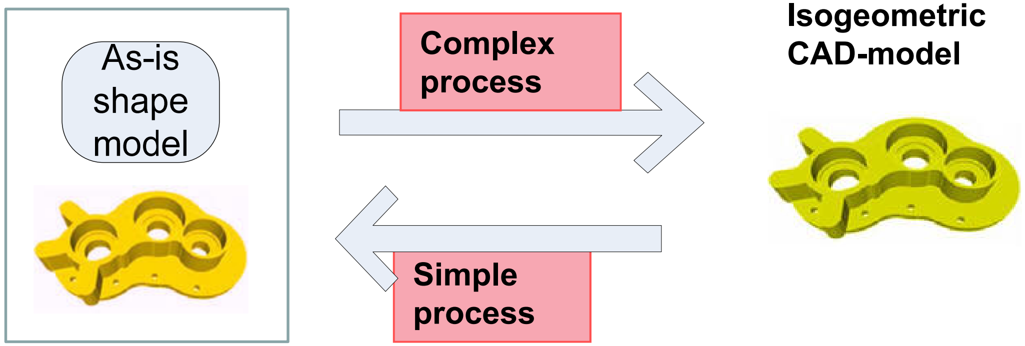
- CAD-models describes the object as planned
  - Combines elementary surfaces (plane, cylinder, cone, sphere, torus and NURBS)
- Models aimed at visual purpose most often represent shape by (texture mapped) triangulations
- Laser scanning efficiently produce millions of points on the geometry
  - Extracting information from 3D datasets is complex
  - A industry is established related to model building from laser scans
  - Using the datasets for validation and updating of 3D models (CAD) is challenging
- The projects following projects address these issue partly using LR B-splines:
  - “3D Airports for Remotely Operated Towers” in SESAR JU (EU)
  - The STREP TERRIFIC – EU ICT-program Factory of the Future
  - New IP- IQmulus on large GIS data sets (currently under contract negotiations) (October 2012-September 2016)

LR B-spline representation of area around Svolvær airport, Norway



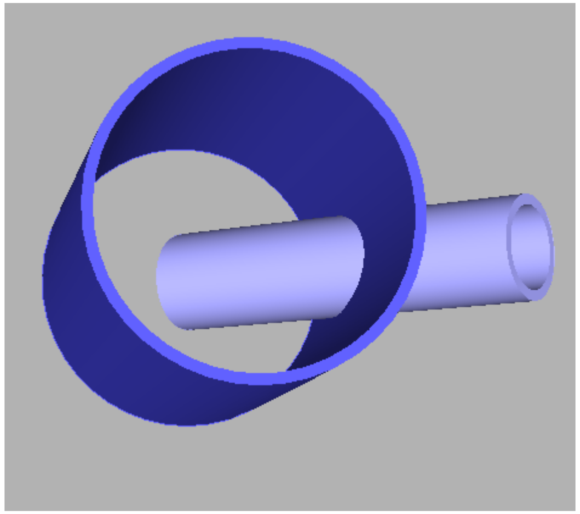
Data courtesy of AVINOR
Model by:
Odd Andersen, SINTEF

Challenge 2: Create 3-variate isogeometric model

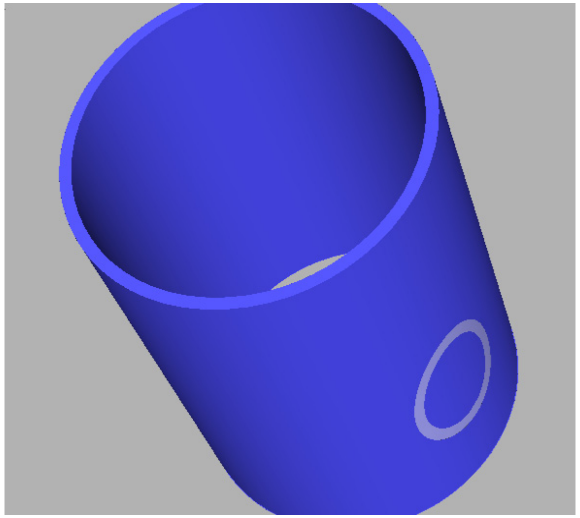


- The "As-is" shape model describes mathematically only the inner and outer hulls (surfaces) of the object using triangulations, elementary surfaces or NURBS surfaces.
- The isogeometric model is analysis/simulation suitable and describes the volumes mathematically by watertight structures of blocks of 3-variate rational splines
- Building an isogeometric model is a challenge:
  - There is a mismatch between the surface patch structure of the "As-is" model, and a suited block structure of an Isogeometric 3-variate rational spline model.
  - Augmented spline technology is needed such as the novel Locally Refined Splines.
- Projects addressing this
  - Isogeometry (2008-2012)- KMB project funded by the Norwegian Research council
  - TERRIFIC (2011-2014) – STREP funded by the EU ICT

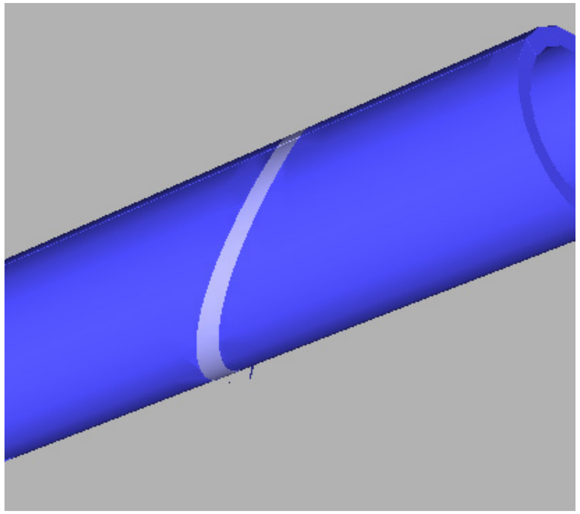
Example: Isogeometric tube joint - Intersection



- Two independent pipes coming from CAD and described as 3-variate volumes



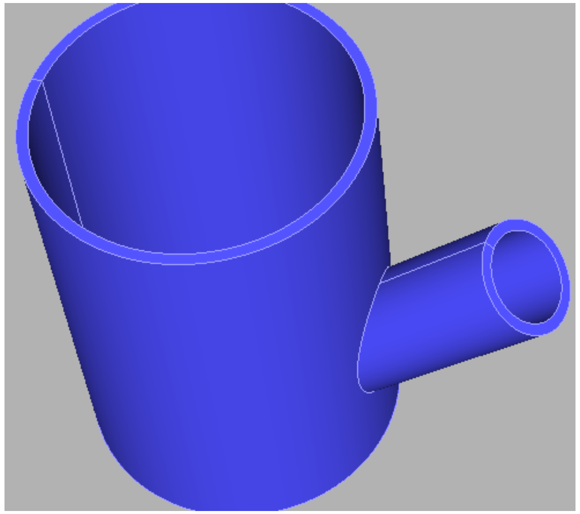
- The intersection of the pipes calculated.
- The original large pipe is split in 3 volumes

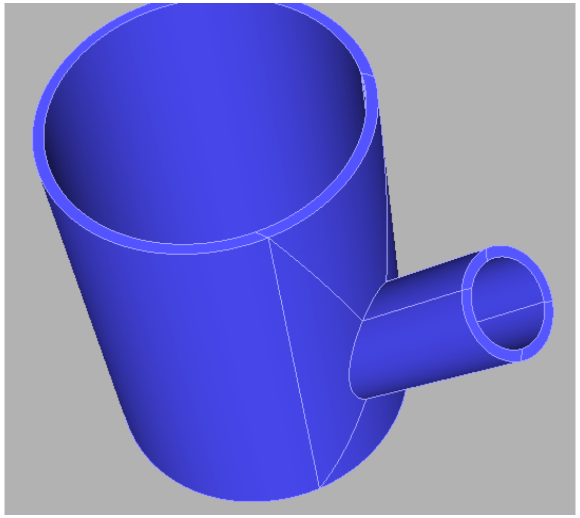


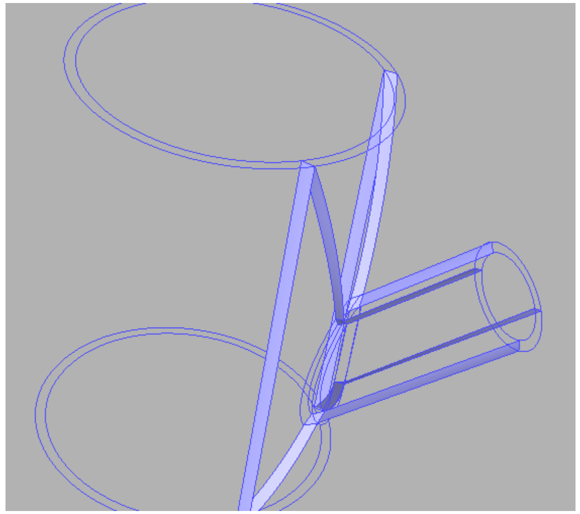
- The intersection of the pipes calculated.
- The original small pipe is split in 3 volumes

Example by:
Vibeke Skytt,
SINTEF ICT

Example: Isogeometric tube joint – Composing volumes







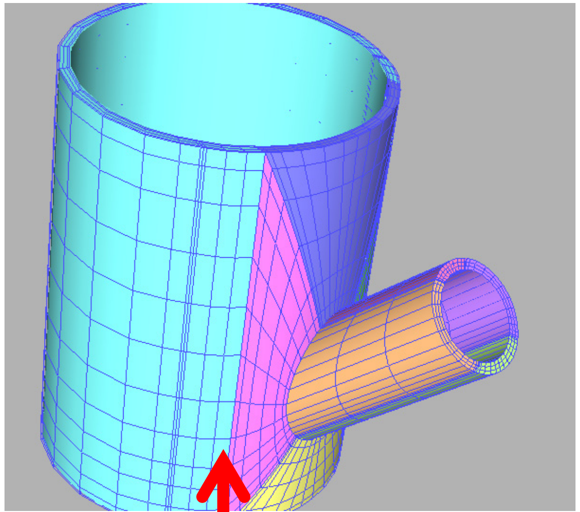
- The relations between the sub volumes produced by the intersection are established
- These volumes do not satisfy the hexahedral (box structure) of the need isogeometric sub volumes

- The volumes split to produce hexahedral volumes

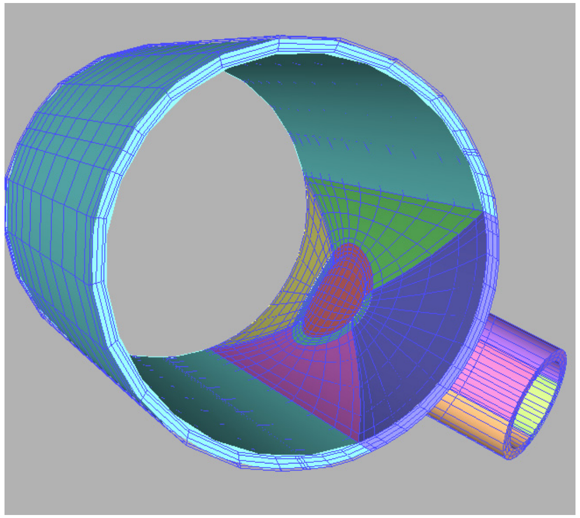
- The internal faces produced by the splitting process

Example by:
Vibeke Skytt,
SINTEF ICT

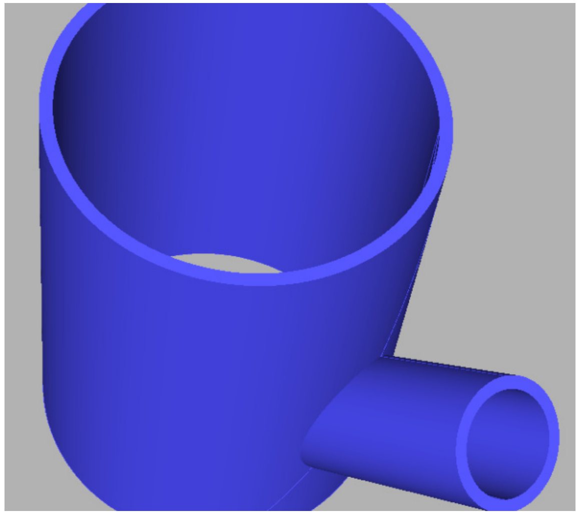
Example: Isogeometric tube joint – match spline spaces(B-splines)



- Spline space refined to have matching lines in each hexahedral NURBS-block to produce a watertight representation



- Same as to the left, different view



- The final isogeometric tube joint.

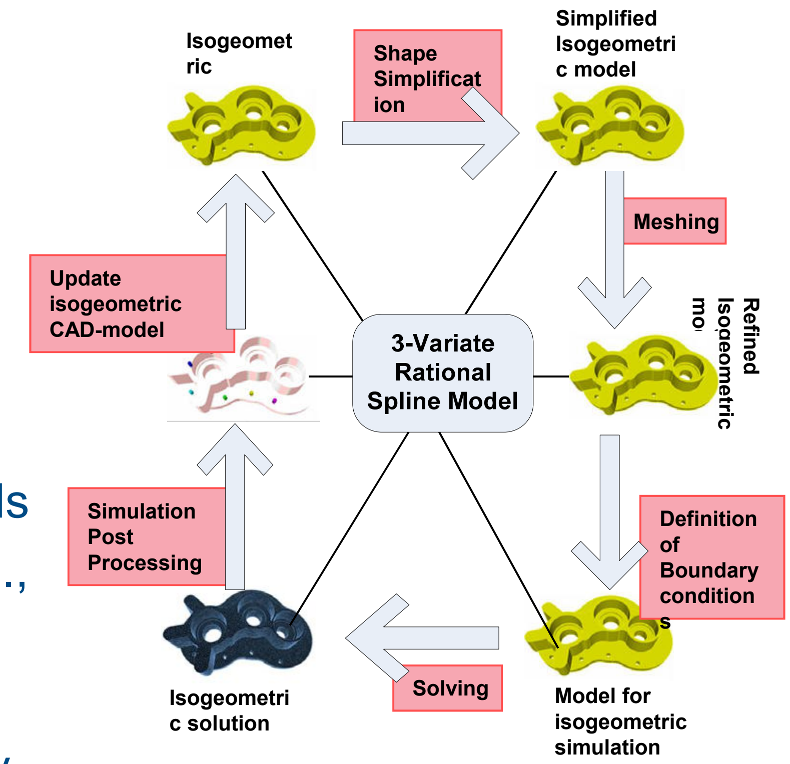
Matching spline spaces of adjacent tensor product B-splines volumes drastically increases the number of vertices. (Local refinement needed)

Example by:
Vibeke Skytt,
SINTEF ICT

Challenge 3: Isogeometric analysis

First introduced in 2005 by T.J.R. Hughes

- Replace traditional Finite Elements by NURBS - NonUniform Rational B-splines
- Accurate representation of shape
- Allows higher order methods
- Perform much better that traditional Finite Elements on benchmarks
- Refinement of analysis models without remeshing
- Exact coupling of stationary and rotating grids
- Augmented spline technology is needed, e.g., Locally Refined Splines

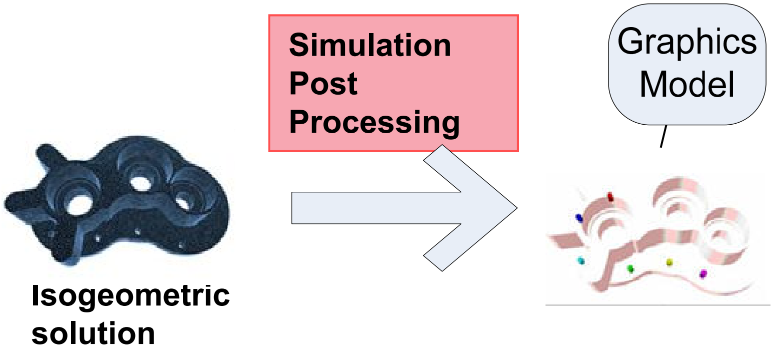


Projects:

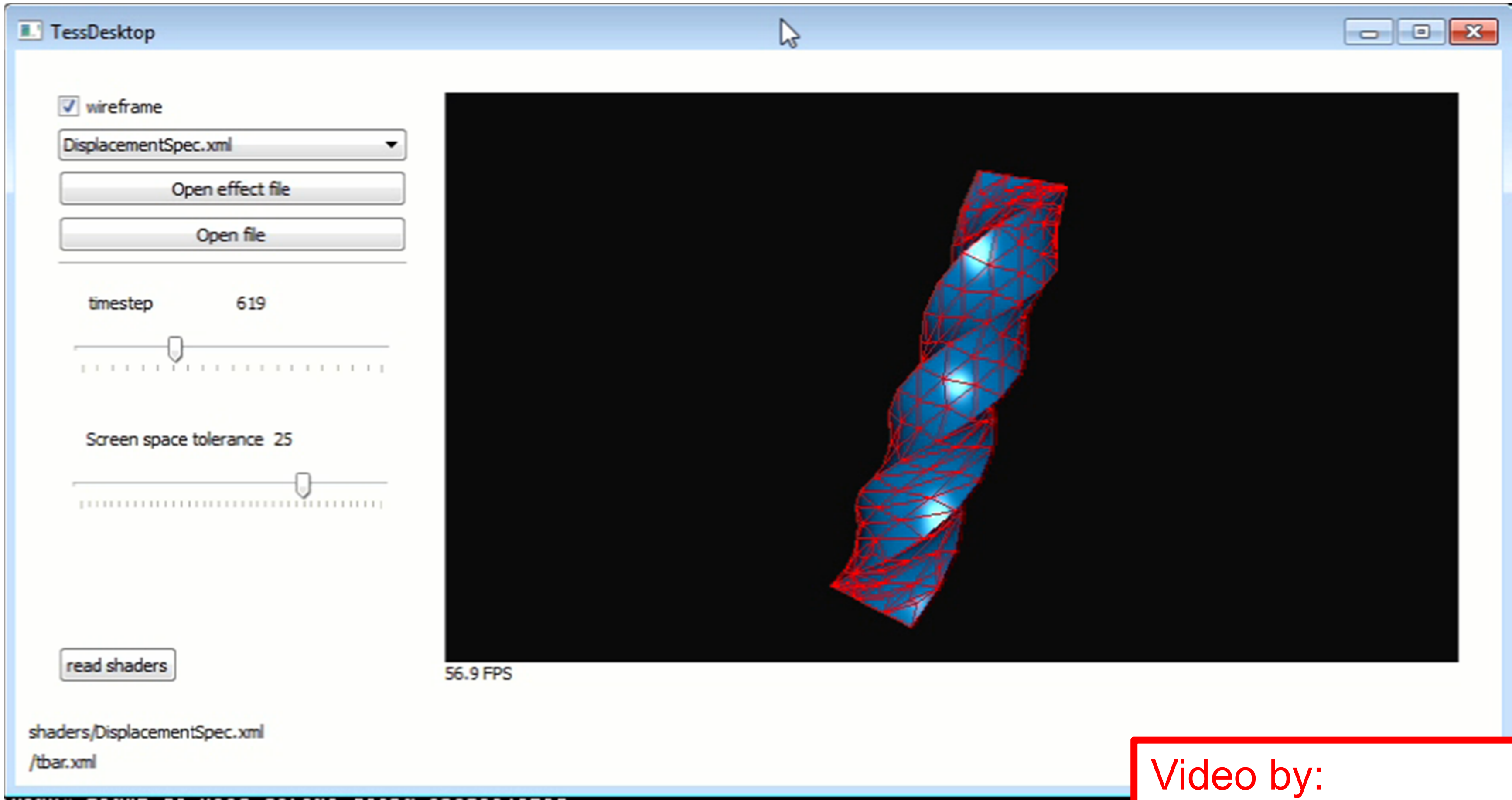
- ICADA (2009-2014)– KMB Project funded by Norwegian Research Council and Statoil
- Exciting (2008-2012)
- TERRIFIC (2011-2014) – STREP EU ICT-program

Challenge 4: Isogeometric visualization

- Traditional visualization technology is triangle based (tesselation)
  - The isogeometric model has to be approximated with triangles for visualization
  - Results are degraded and information lost
- Need for visualization solutions exploiting the higher order representations
  - Higher order representations are more advanced and can better represent singularities in the solution
  - Create view dependent tessellation of splines on the GPU
  - Direct ray tracing on the GPU:
- Project: Cloudviz (2011-2014)



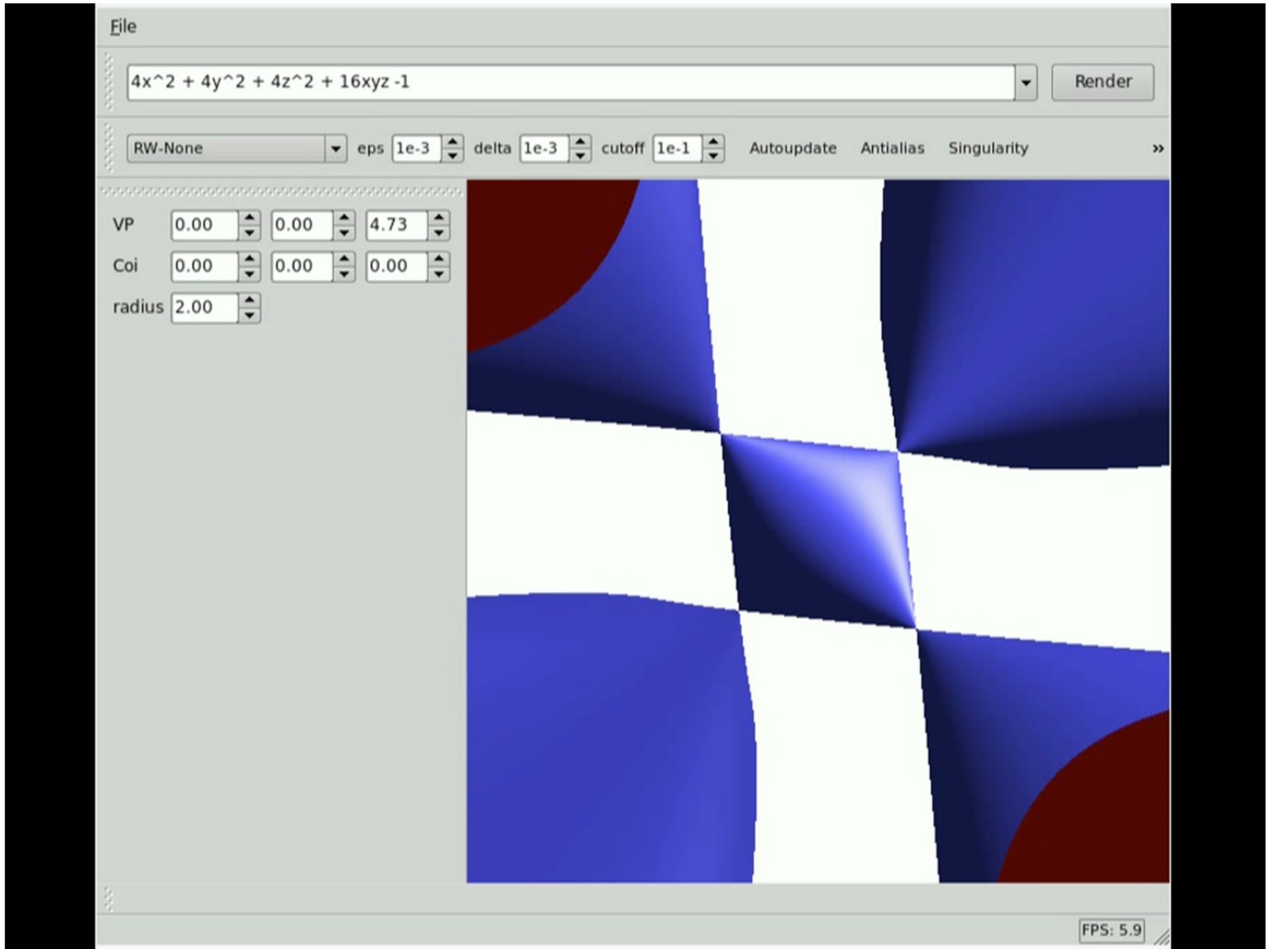
Isogeometric view dependent tesselation of the spline model



Video by:
Jon Hjelmervik,
SINTEF ICT

Click her for video:
http://www.youtube.com/watch?v=KOsDBx8yEt0&list=UU_GWvrs307jzpjIWvQxWwHA&index=1&feature=plcp

Direct ray-casting on the GPU



Video by:
Johan S. Seland,
SINTEF ICT

Click her for video: http://www.youtube.com/watch?v=ORFhU3diakA

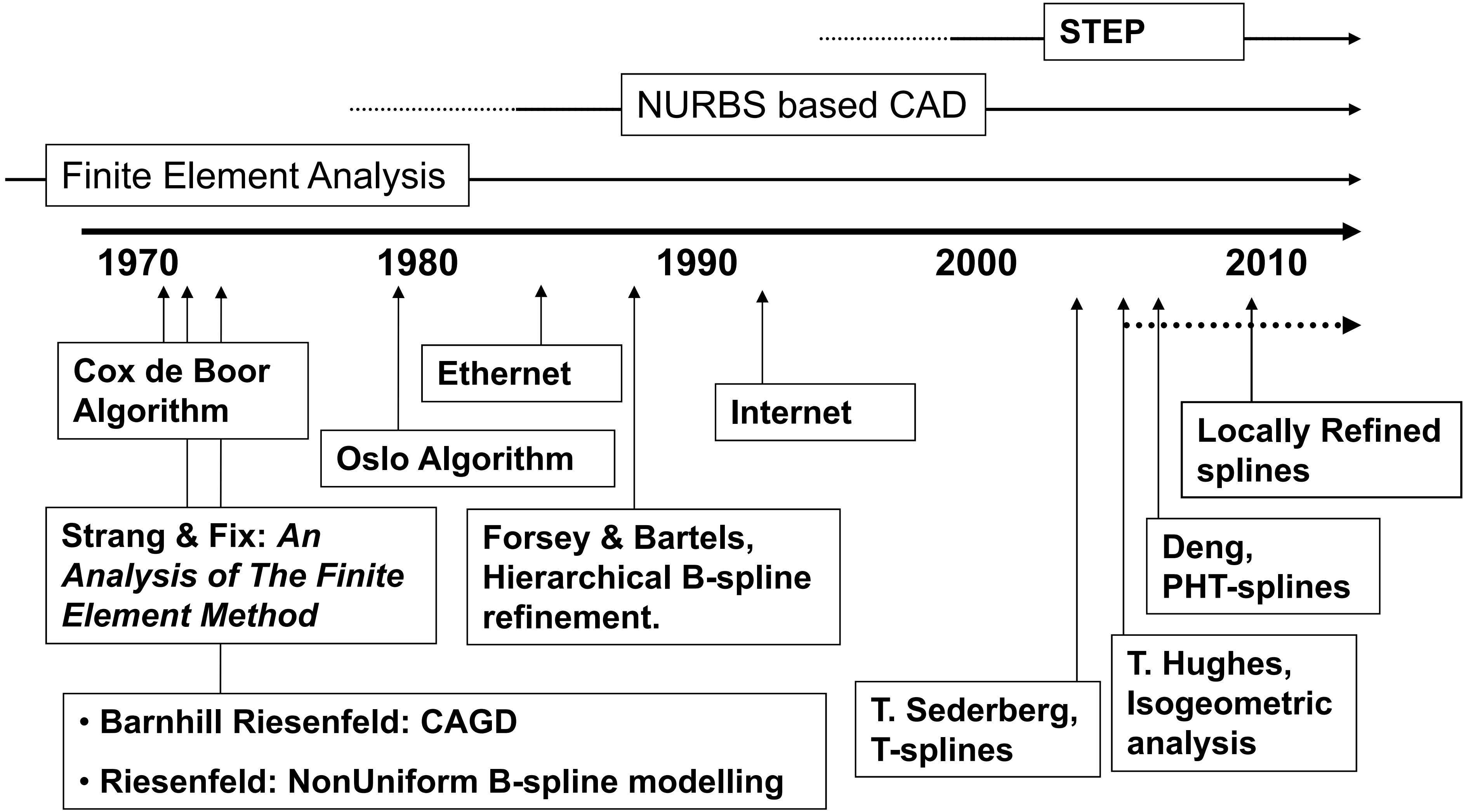
From stand alone computers and systems to integrated information flows

- As long as communication between computers was hard, information exchange remained paper based
  - The Ethernet invented by Xerox Parc in 1973-1975,
  - ISO/IEEE 802/3 standard in 1984
  - Deployment in industry started, simple communication between computers
- CAD Data Exchange introduced
  - IGES - *Initial Graphics Exchange Specification*

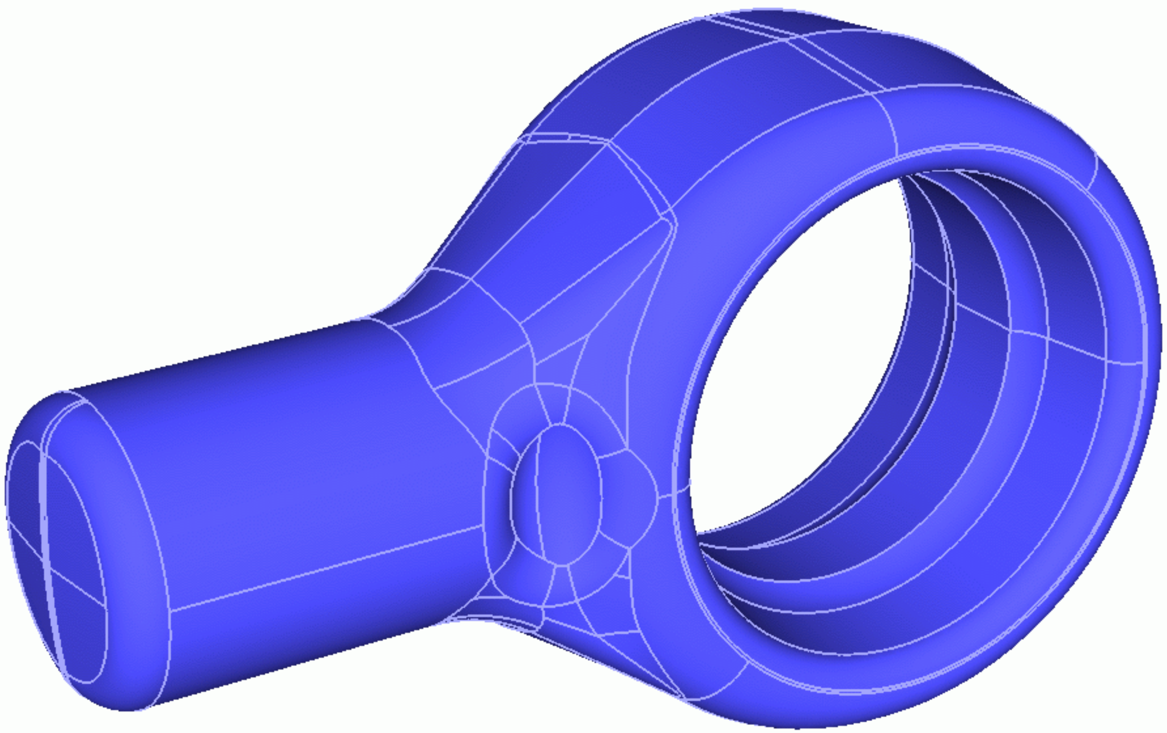
    Version 1.0 in 1980
  - STEP - *Standard for the Exchange of Product model data*

    started in 1984 as a successor of IGES, SET and VDA-FS, Initial Release in 1994/1995, deployment in industry started
- The Internet opened to all 1991
  - Start of deployment of data exchange between processes over the Internet

Timeline important events related to FEA and Isogeometric Analysis



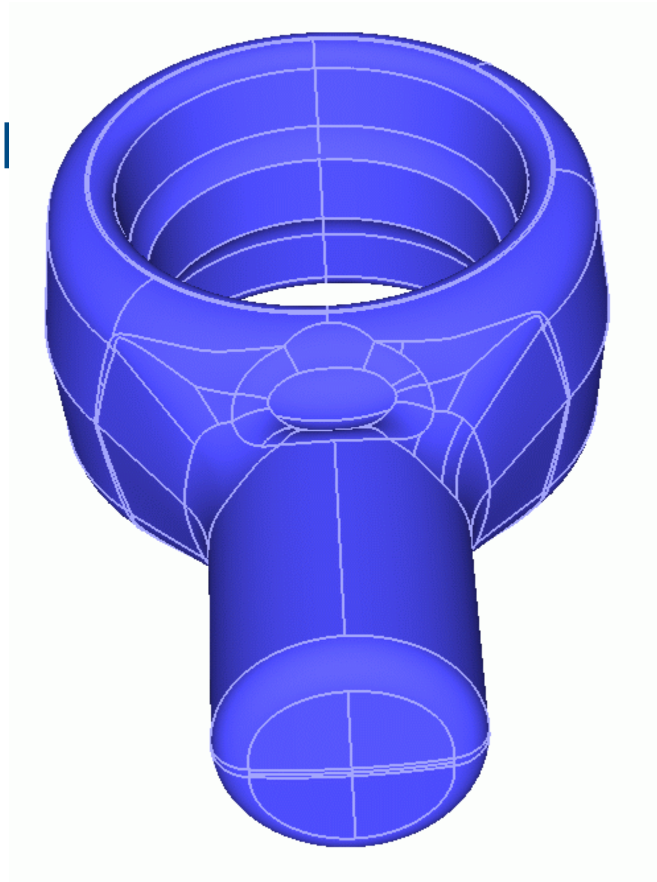
CAD has to change to support isogeometric analysis



- Example: Patch structure of a fairly simple CAD-object
  - Object designed patch by patch to match the desired shape
  - Shape designed for production

CAD patch structure not an obvious guide to isogeometric block structure

- We would like considerably fewer NURBS blocks than the number of surfaces patches in the CAD-model
- The object has three main parts
  - The “torus” like part
  - The “cylindrical” handle
  - The transition between these
- Not obvious how this can be represented as a composition of NURBS blocks
  - Acute angles
  - Extraordinary points
  - Singular points



Current CAD technology is here to stay

- The major part of revenue of CAD vendors comes from industries that don't suffer from the CAD to analysis bottleneck. However, advanced industry suffers.
- Current CAD is standardized in ISO 10303 STEP.
- The driving force for isogeometric CAD has to be industries that has the most to gain from the novel approach, e.g.,
  - aeronautics, defense, space and automotive industries
- Isogeometric CAD: A next natural step in CAD evolution?
- ISO 10303 STEP should also include isogeometric CAD

Two approaches to isogeometric CAD

1. Build the block structure one block at the time
   - User responsible for block interfaces and interfaces to outer and inner hulls.
   - Similar to surface modeling without trimming
   - Can be template based
2. Design the tri-variate block structure in an already existing ISO 10303 STEP type CAD model
   - The user controls the block structure. The blocks snap together and to outer and inner hulls.
   - Similar to designing surfaces into a point cloud in reverse engineering
   - Last week (June 2012) my colleague Vibeke Skytt participated in an ISO meeting in Stockholm to find out how ISO 10303 STEP can better support the idea of isogeometric analysis.

NURBS lack local refinement

- The regular structure of tensor product NURBS does not allow local refinement
- 1988: Forsey & Bartels: Hierarchical B-spline refinement.
  - $f(s,t) = f_1(s,t) + f_2(s,t) + \ldots + f_n(s,t)$
  - The spline space of f_{i+1} is a refinement of the spline space of f_i
- 1998: Rainer Kraft, Adaptive und linear unabhängige multilevel B-splines und ihre Anwendungen. PhD Thesis
- 2003: T. Sederberg, T-splines
  - Compact one level of hierarchical B-splines in the surface control grid. Generalization based on the control grid of B-spline surfaces
- 2006: Deng, PHT-splines
  - C^1 Patchwork of bi-cubic Hermite surface patches allowing T-joints between patches
- 2009: Locally Refined B-splines (LR B-splines), addressing local refinement from the viewpoint of CAGD and Analysis
- 2010 Hierarchical B-splines and IGA – Jüttler, Giannelli, et. al.