IGA @ AG.JKU.AT

IsoGeometric Analysis at Johannes Kepler University

Angelos Mantzaflaris

RICAM-Linz

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C. Giannelli, D. Grossmann, B. Jüttler, M. Kapl, M. Rossgatterer, T. Takacs

Contents of this seminar

- Some words about JKU

1. Hierarchical B-spline bases
   - local refinement, suitability for IGA.
2. Singular parameterizations
   - Treat singular cases, restore approximation order.
3. Turbine modeling
   - Compact turbine CAD model and Volume parametrization.
4. Turbine simulation
   - Simulation & comparison to FEA in industrial problems.

Johannes Kepler University Linz (JKU)

- Leading university of Upper Austria, which is one of the most dynamic and successful regions in Austria.
- Around 17,000 students with 1 in 11 being from abroad.

Johannes Kepler University Linz (JKU)

- Named after the famous astronomer Johannes Kepler who lived in Linz.
- Faculty of Engineering and Natural Sciences: strong links to the local economy.

Johannes Kepler University Linz (JKU)

- Institute of Applied Geometry:
  1 of the 10 math institutes of JKU.
  Established in 2000*, in conjunction with the appointment of Bert Jüttler as prof. of Scientific Computing.*

- Current staff: 1 professor, 3 postdocs, 7 PhD students

- Collocated with the RICAM research institute of the Austrian Academy of Sciences.

Johannes Kepler University Linz (JKU)

Institut für Angewandte Geometrie

- The research activities focus on applications of geometry
  - CAGD - Computer Aided Geometric Design
  - Applications of (Classical) Algebraic Geometry
  - Kinematics and Robotics
  - Isogeometric Analysis

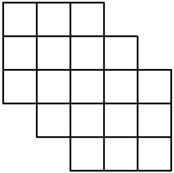
1. Hierarchical B-spline bases
    - local refinement, suitability for IGA.
2. Singular parameterizations
    - Treat singular cases, smooth subspaces.
3. Turbine modeling
    - Compact turbine CAD model and Volume parametrization.
4. Turbine simulation
    - Simulation & comparison to FEA in industrial problems.

Hierarchical B-Spline Spaces

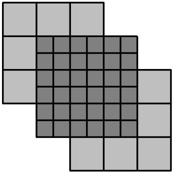
Question studied

Find suitable B-Spline spaces that ...

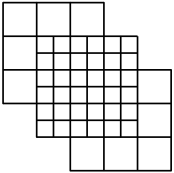
➢ *enable local refinement*



initial grid



marked domain



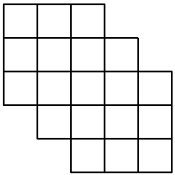
locally refined mesh

➢ *exhibit “nice” properties*

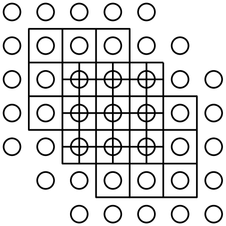
- linear independence, high regularity
- span the spline space, partition of unity, stability

Hierarchical splines: background

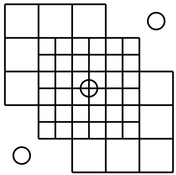
Approach: hierarchical B–spline model



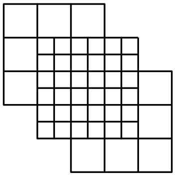
level 1



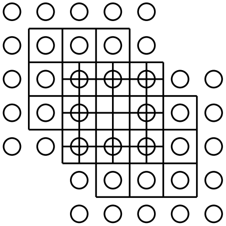
overlapping B-splines



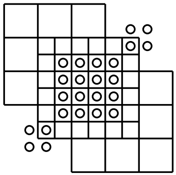
non–selected B-splines



level 1& 2

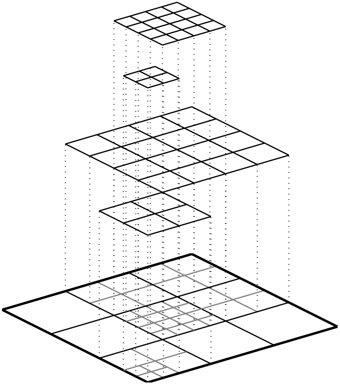


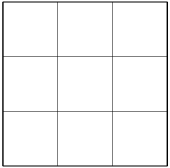
B–splines of level 1



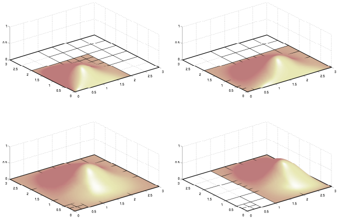
B–splines of level 2

Hierarchical splines: basis functions

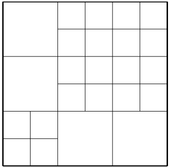




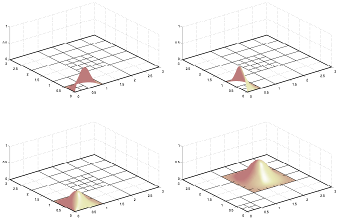
domain of level 1



some B–splines of level 1



domains of level 1&2



some B–splines of level 2

Hierarchical splines: results

Properties of hierarchical B–splines:

[✓] local refinement

[✓] linear independence

[✓] nested spaces

[✓] maximum regularity

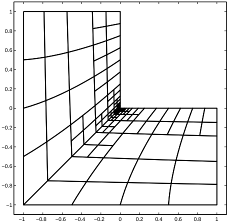
[✓] weak stability

Questions:

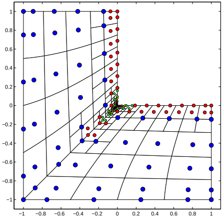
- suitable for IGA?
- characterization of the spline space?
- partition of unity?
- Stability?

Hierarchical refinement in IGA

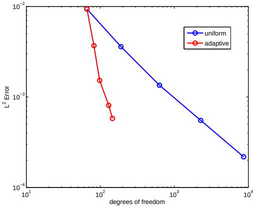
- Is it advantageous in the context of IGA?
- Yes, greatly reduces DoFs.
  Example: heat equation over L-Shape:



refined grid



hierarchical B–splines



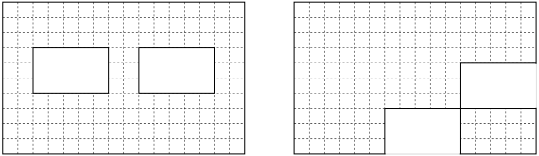
convergence plot

[Vuong, Giannelli, Jüttler, Simeon (CMAME, 2011)]

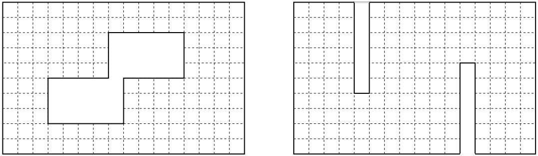
Hierarchical splines: spaces

Characterization of the underlying spline space

- Contain piecewise polynomials of degree d and smoothness C^{d-1}?
- yes, under reasonable assumptions on the domain configuration.



admissible domain configuration for degree 2



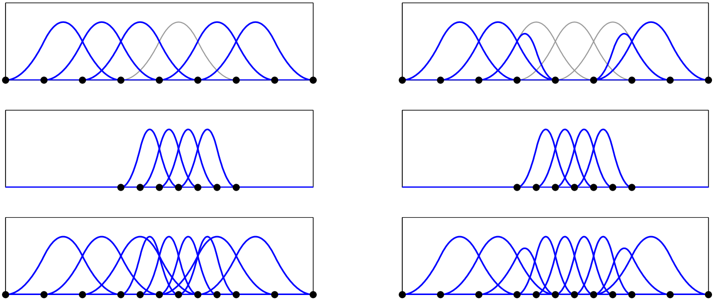
non–admissible domain configuration for degree 2

[Giannelli, Jüttler (DK–JKU report, 2011)]

Hierarchical splines: normalization

Truncated hierarchical B–splines

- Partition of unity?
- yes, by truncated splines (also, reduce support overlap).



hierarchical truncated

[Giannelli, Jüttler, Speleers (CAGD, 2012)]

Hierarchical splines: stability

Weak/strong stability of multi–level basis

- Is it stable?
- yes, in fact truncated HB-splines are strongly stable.

$$f(u,v) = \sum c_\tau \tau \qquad C = \{c_\tau\} \qquad k_0 ||C||_\infty \leq ||f(u,v)||_\infty \leq k_1 ||C||_\infty$$

$\ell = \#$ of levels

- hierarchical B–splines:
  - k_0 and k_1 depend polynomially on $\ell \Rightarrow$ weakly stable basis

[Kraft (PhD thesis, 1998)]

- truncated hierarchical B–splines:
  - k_0 and k_1 do not depend on $\ell \Rightarrow$ strongly stable basis

[Giannelli, Jüttler, Speleers (preprint, 2012)]

Singular Parameterizations

Singular parametrizations: Questions

In case of singularities in the parameterization ...

➢ *determine smooth-enough test functions for numerical simulation*

- classify different cases
- determine regularity properties of the test functions

➢ *keep approximation power (as far as possible)*

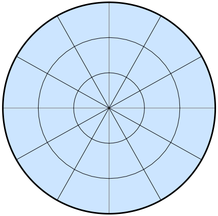
- modify (basis of) test function space to obtain the desired regularity
- analyze approximation power of the modified discretization space

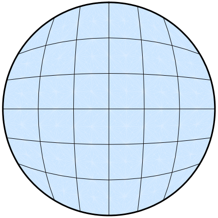
➢ *avoid trimming/splitting*

- Treat singularities in single-patch parametrizations

Singular parametrizations: Motivation

Singular parametrizations arise in ...





- Single-patch parametrization of domains with smooth boundary

  $\rightarrow$ using polar coordinates

  $\rightarrow$ using Coon's patch method

- sharp features (0° angles)
- cones
- etc.

Singular parametrizations: Regularity considerations

- Regularity properties of test functions $\beta_{\mathbf{i}}$ in the presence of singularities?
- There is a smooth subspace of H^ℓ test functions.

Regularity condition:

$$\beta_{\mathbf{i}} = b_{\mathbf{i}} \circ \mathbf{G}^{-1} \in H^\ell(\Omega)$$

where $\mathcal{V}_h = \mathrm{span}\{\beta_i\}$.

Does not always hold if the parametrization is singular, eg.:

Space correction

- Is there a better space for the discretization?
- yes, $\mathcal{V}_h \cap H^\ell(\Omega)$.

$\mathcal{V}_h = \text{span}\{\beta_i\}$ and Ω is the physical domain.

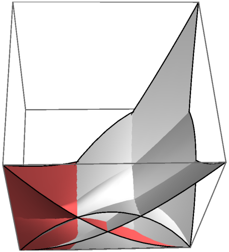
Compute a basis for the restricted test function space

$$\mathcal{V}_h \cap H^\ell(\Omega) \quad \text{for } \ell = 1, 2.$$

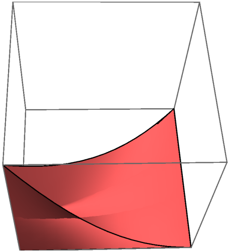
- **idea:** Modify β_i's to get a basis for $\mathcal{V}_h \cap H^\ell$.
- New basis functions are linear combinations of the old ones.
- New basis exhibits nice properties:
  non-negativity, partition of unity, relatively simple construction
- Approximation order ?

[Takacs, Jüttler (CMAME, 2011)]

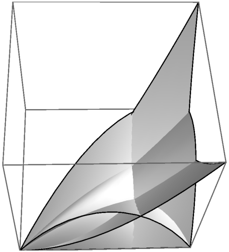
Space correction: H^1 example (triangular domain)



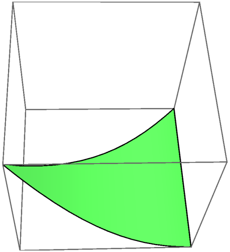
$\mathcal{V}_h$ (dim= 9)



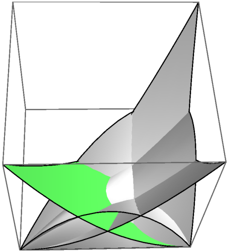
3 test functions $\notin H^1$



test functions $\in H^1$

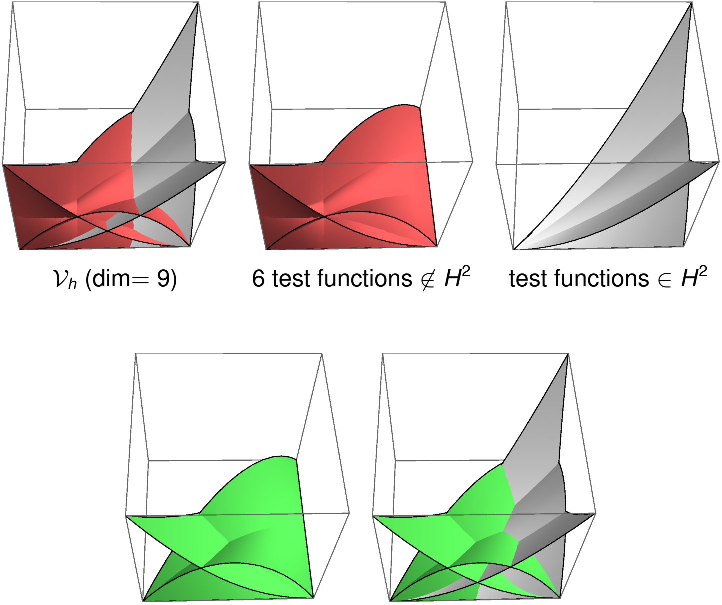


new function $\in H^1$



$\mathcal{V}_h \cap H^1$ (dim= 7)

Space correction: H^2 example (triangular domain)



$\mathcal{V}_h$ (dim$=9$)   6 test functions $\notin H^2$ test functions $\in H^2$

new functions $\in H^2$ $\mathcal{V}_h \cap H^2$ (dim$=6$)

Approximation power of $\mathcal{V}_h \cap H^\ell$

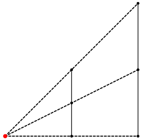
What are the approximation properties of $\mathcal{V}_h \cap H^\ell$?

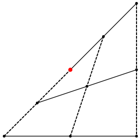
- In regular B-Spline parametrizations the error is $O(h^{d-\ell+1})$.
- We need to bound

$$\inf_{\varphi_h \in \mathcal{V}_h \cap H^\ell} \|\varphi - \varphi_h\|_{H^\ell(\Omega)} \leq \ ?$$

  for any $\varphi \in H^\ell$ (ie. solution of a boundary value problem).

- Near-optimal approximation order:





	approx. order	approx. order
L^2 :	$d+1$	$d+1$
$\mathcal{H}^1$:	d	$d-1/2$
$\mathcal{H}^2$:	$d-1$	$d-5/2$

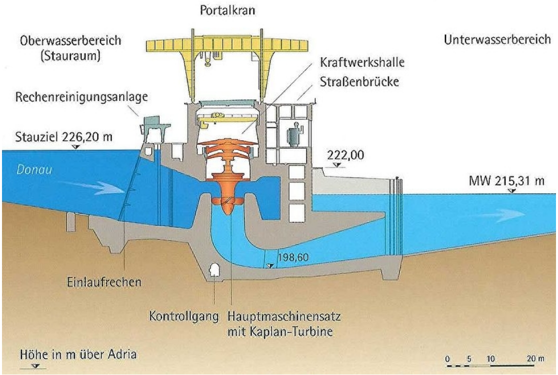
[Takacs, Jüttler (Graph. Models, 2012)]

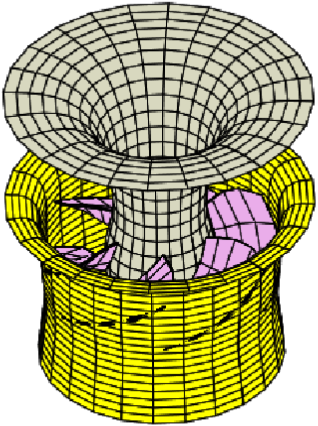
Turbine modeling

Questions studied

- Compact CAD-model for a turbine blade?
- Volume parameterization of the water passage?

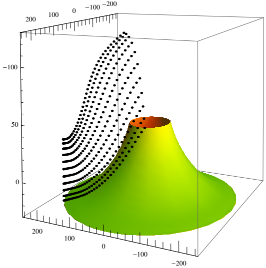
Motivation: EXCITING project (partner HYDRO: hydroelectric turbines)



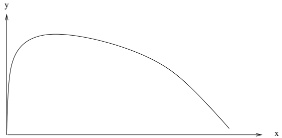


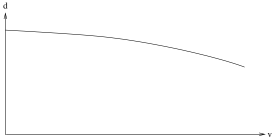
Description of a turbine blade

- **Input :**
  - medial surface



  - design parameters: profile function (shape of cross sections) and scaling function (thickness of blade)





- **Output:** B-spline model of the blade

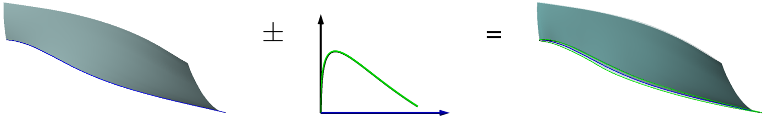
Description of a turbine blade

- Pressure and suction side of the blade are obtained by adding or subtracting the scaled profile to the medial surface:

$$\mathbf{b} = \mathbf{m} \pm d\mathbf{n}y$$

$\mathbf{m}$ . . . . . medial surface
$\mathbf{n}$ . . . . . normals of the medial surface
y . . . . . profile function
d . . . . . scaling function

- Schematic computation



Constructing a B-spline representation

Getting the boundary B-Spline surfaces of pressure and suction sides.

- **Method:**
  1. B-spline representation of medial surface
  2. Reparameterization of the medial surface
  3. Computation of the normals
  4. Generating the blade surface

[Rossgatterer, Jüttler, Kapl, Della Vecchia (Graph. Models, 2012)]

Medial surface, normals, reparameterization

- Approximate medial surface $\mathbf{m}(u, v)$ by B-spline surface
  $\rightarrow$ least-squares fitting
- Approximate normals of medial surface $\mathbf{n}(u, v)$ by B-splines
  $\rightarrow$ least-squares fitting
- Reparameterize according to profile:
  Horizontal speed of profile = parametric speed of medial surface

- Look for a function $u(t)$ s.t.

$$\|\frac{\partial}{\partial t}\mathbf{m}(u(t), v)\| = \lambda(v)x'(t)$$

  - $\lambda(v)$: arc length of u-streamline of $\mathbf{m}(u, v)$ at v.
  - x : first coordinate of the profile curve.
- Minimization problem $\Rightarrow$ Gauss-Newton-type method.

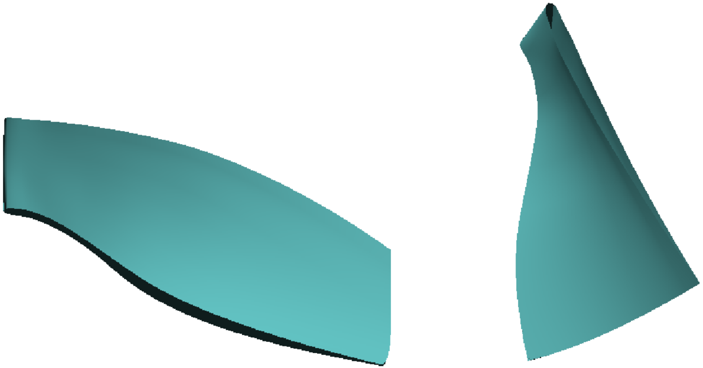
Generating the blade surface

- The two sides of the blade (pressure and suction side) are obtained by

$$\mathbf{b}^{(\pm)}(t,v) = \mathbf{m}(u(t),v) \pm d(v)\mathbf{n}(u(t),v)y(t)$$

- Can compute $\mathbf{b}^{(\pm)}$ by using formulas for the composition and multiplication of B-spline functions.
- We arrive to a parametric model of the turbine blade.
- $\mathbf{b}^{(+)}$ and $\mathbf{b}^{(-)}$ fit together at $t = 0$ with C^1-continuity.
- High number of control points and high order
  $\rightarrow$ degree reduction and knot removal
  $\rightarrow$ coarse surface representation

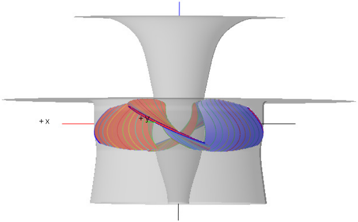
Example - Turbine blade

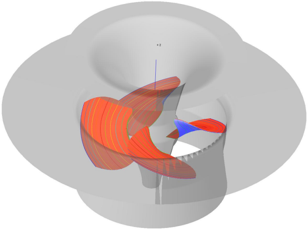


Volume parameterization of water passage

Getting NURBS-volume parameterization of domain where water flows.

Kaplan turbine with 4 blades, mostly used in river power stations



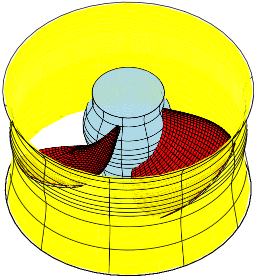


- Water passage is to be meshed for simulation & optimization

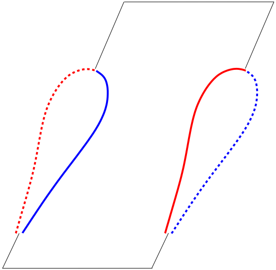
Input data

- Problem is symmetric around the turbine
  → restrict to passage between two consecutive blades

  Bounded by suction & pressure side, inlet and outlet



domain of interest

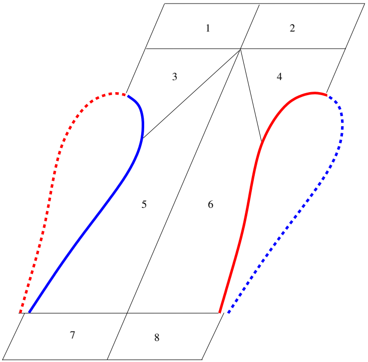


cylindric projection

1. Divide domain of interest into volume patches
2. Parameterize each volume patch individually
3. Fit volumes together with C^0-continuity

Segmentation

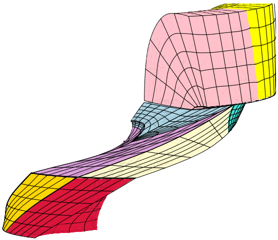
- There exist many ways to subdivide the domain.
  $\longrightarrow$ avoid singularities and too large or small angles



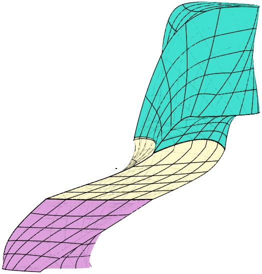
- Segmentation into 8 patches, each of them being topologically equivalent to the cube.

Example - Volume

➢ Final volume parameterization of (a sixth of) the water passage



8 patches



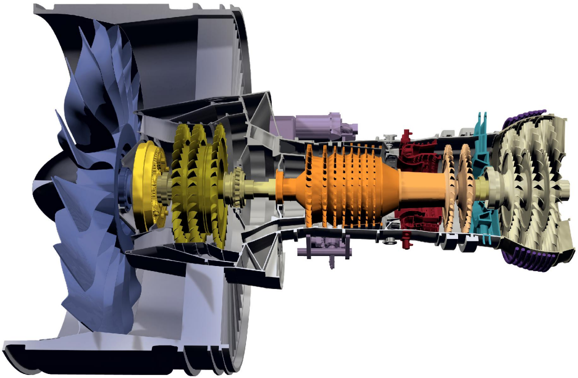
3 patches

- Generated B-spline parameterizations of blades and volumes that are suitable for Isogeometric Analysis.

Turbine Simulation

Simulation of turbine engines

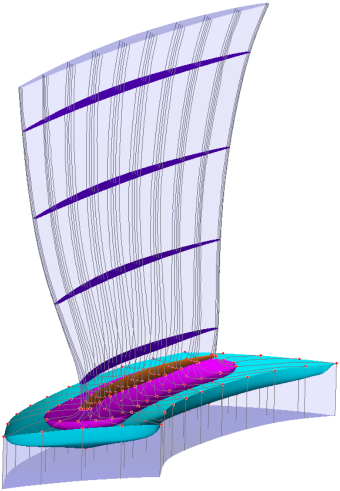
Apply IGA in a real industrial environment, compare with FEA



MTU Aero Engines

Parameterization of a blade volume

B-Spline model of a turbine blade, closer look



numerical simulation

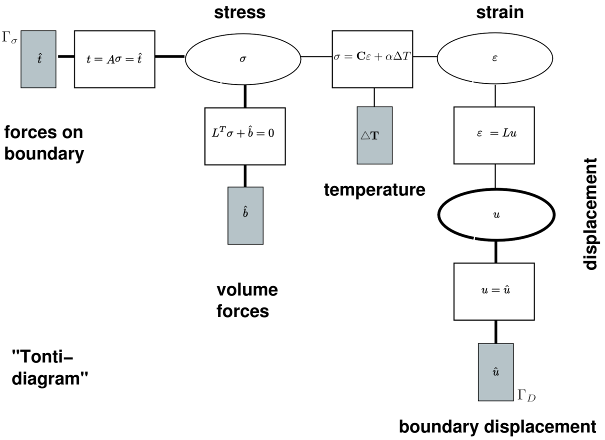
Deformation of turbine blades, linear elasticity assumption

- Structural simulation subject to pressure and temperature field
- Based on single-patch isogeometric solver produced by TU Munich (EXCITING project)
- Input (pressure field, temperature field) generated by other departments of MTU
- Comparison with results generated by the standard FEM solver CalculiX

[Grossmann, Jüttler, Schlusnus, Barner, Vuong (CAGD, 2012)]

mathematical model

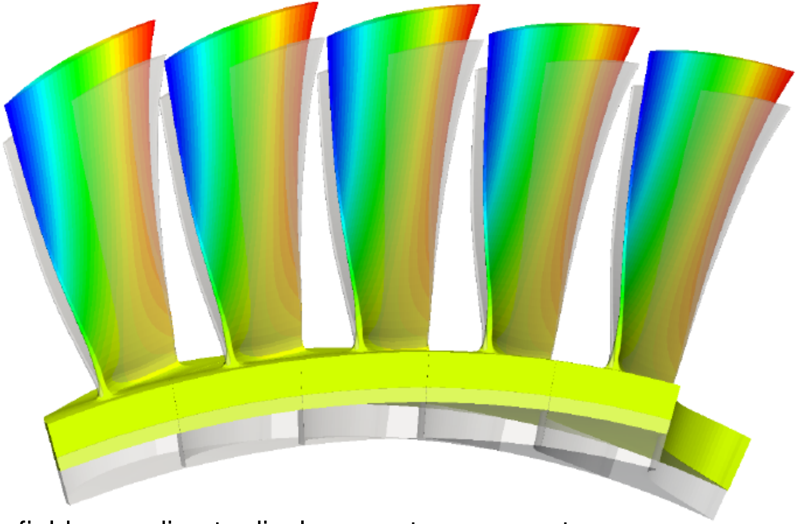
linear elasticity with temperature-dependent material properties:



- Blades take intended shape under the influence of high temperature, pressure, and centrifugal forces

comparison IGA / standard FEM (CalculiX)

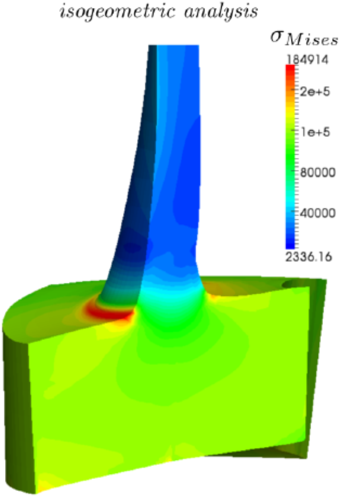
Blade deformation under centrifugal force.



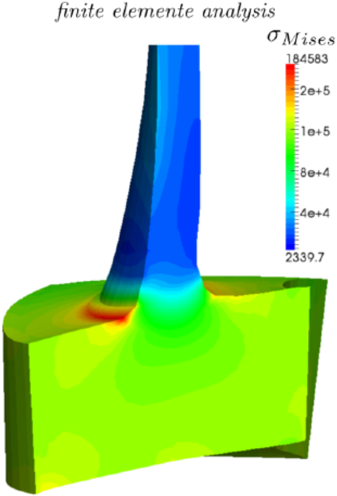
Color field according to displacement component u_{φ}.

comparison IGA / standard FEM

Stress analysis (sliced blade)



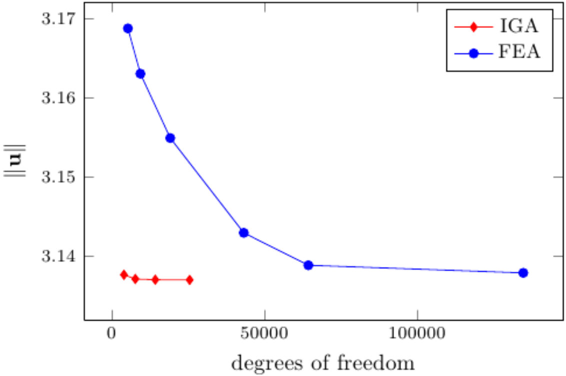
DoFs = 4004



DoFs = 43228

comparison IGA / standard FEM

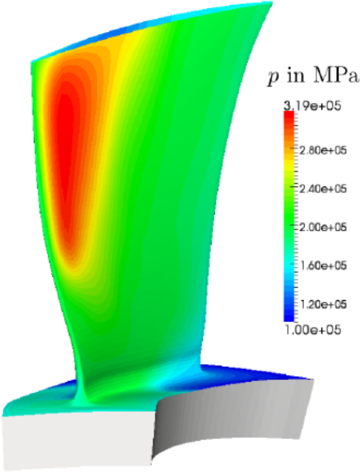
Convergence behavior



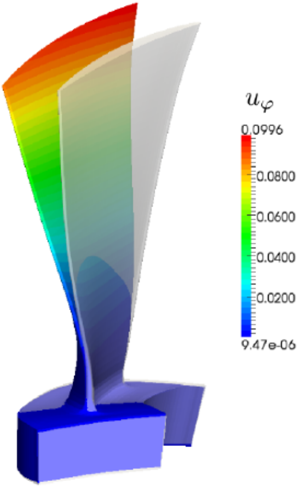
$\|\boldsymbol{u}\|$: displacement at top corner of blade under centrifugal force.

comparison IGA / standard FEM

Pressure on the suction side of the airfoil

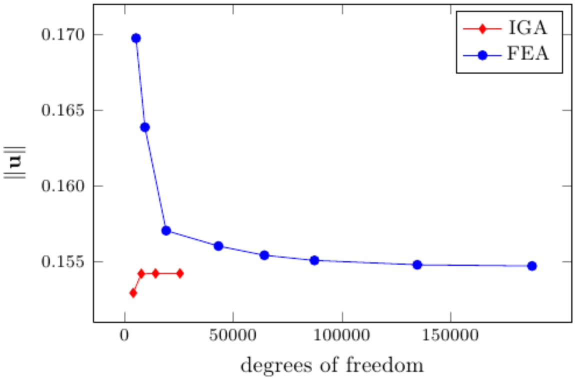


Deformed shape (color: disp. component u_φ)



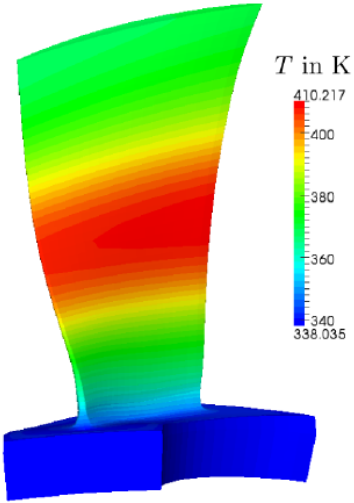
comparison IGA / standard FEM

Displacement on top corner caused by surface pressure

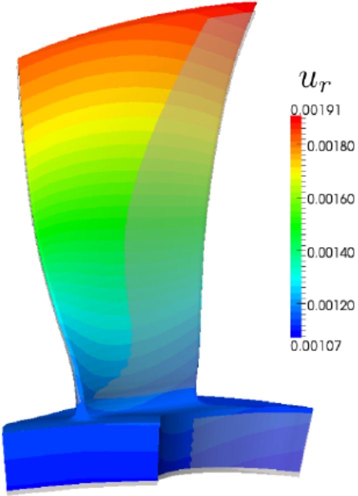


comparison IGA / standard FEM

Temperature distribution

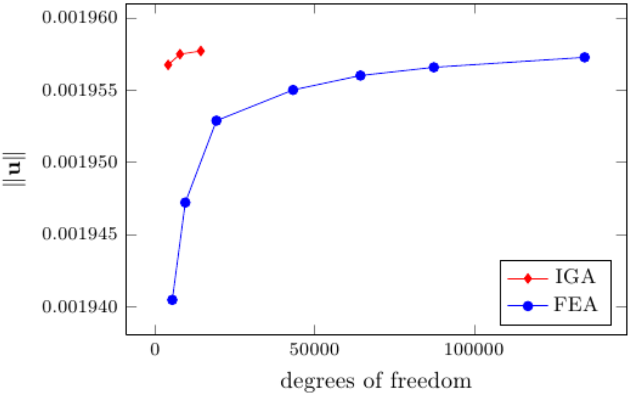


Resulting displacement



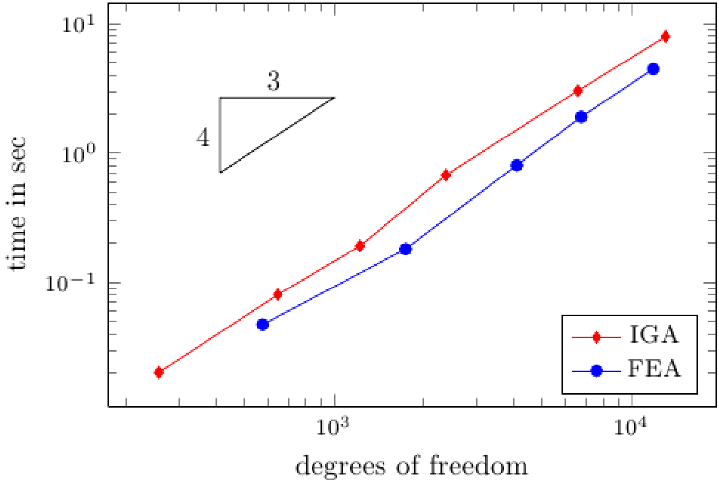
comparison IGA / standard FEM

Displacement at the top corner caused by temperature distribution



comparison IGA / standard FEM

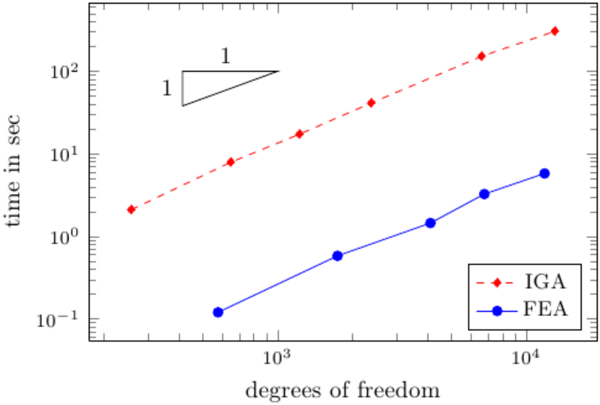
Time required to solve the linear system.



comparison IGA / standard FEM

Time required for stiffness matrix assembly.

(code used for IGA was not optimized)



Summing up

- Specialized bases for use in IGA
  - Hierarchical B-Splines
    (local refinement, suitable for IGA)
  - Singularity analysis and smooth subspaces
    (treat singular points, near-optimal approximation order)
- Applying IGA to blade modeling and simulation
  - Constructing the CAD model & volume
    (compact parametrizations, respect design specifications)
  - Simulation and comparison to FEA
    (show the advantage of IGA in an industrial setting)