Isogeometric Analysis for Shell Structures

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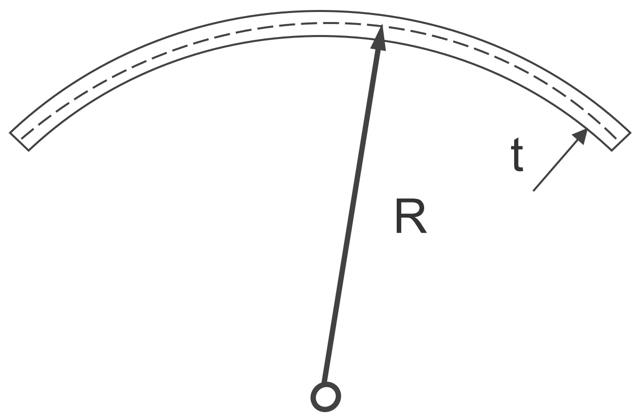
Isogeometric Analysis for Shell Structures

Outline

- Introduction to shells
- Kirchhoff-Love shell theory
- Isogeometric shell analysis
- Integration into CAD
- Application to FSI simulations of wind turbine blades
- Isogeometric shape optimization

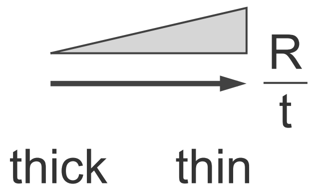
Introduction – What is a Shell?

A shell is a curved three-dimensional, thin-walled structure. It can be represented by a curved surface with a certain thickness.



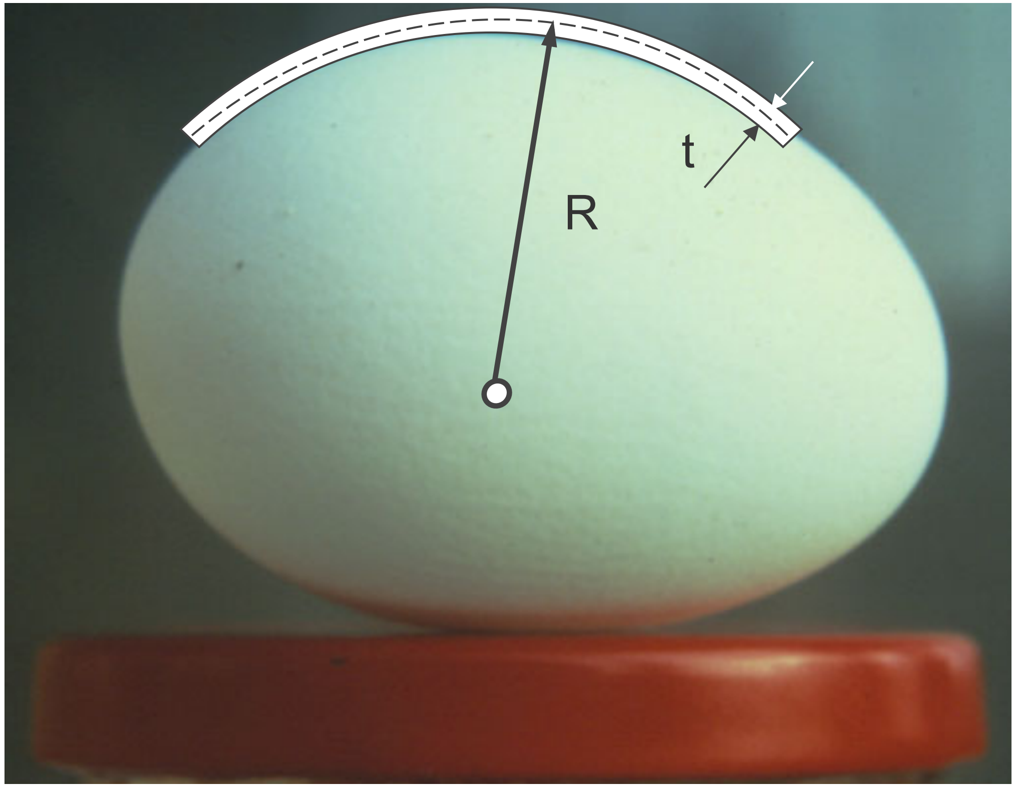
Due to the curvature, shells can carry the load mostly by membrane forces, reducing bending moments. This implies a very efficient use of the material and allows to be very thin and light.

$$\text{slenderness} = \frac{\text{radius of curvature}}{\text{thickness}} = \frac{R}{t}$$

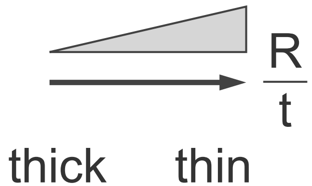


Shells in nature

Example:
The Egg



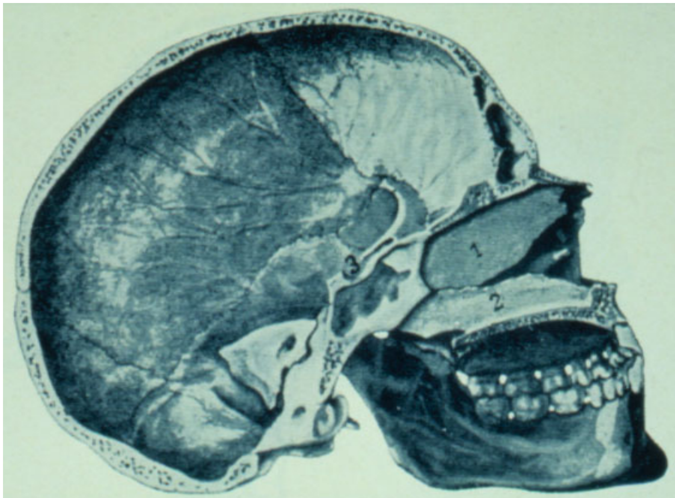
$$\text{slenderness} = \frac{\text{radius of curvature}}{\text{thickness}} = \frac{R}{t} \cong 60$$



Shells in nature

seashell

nutshell



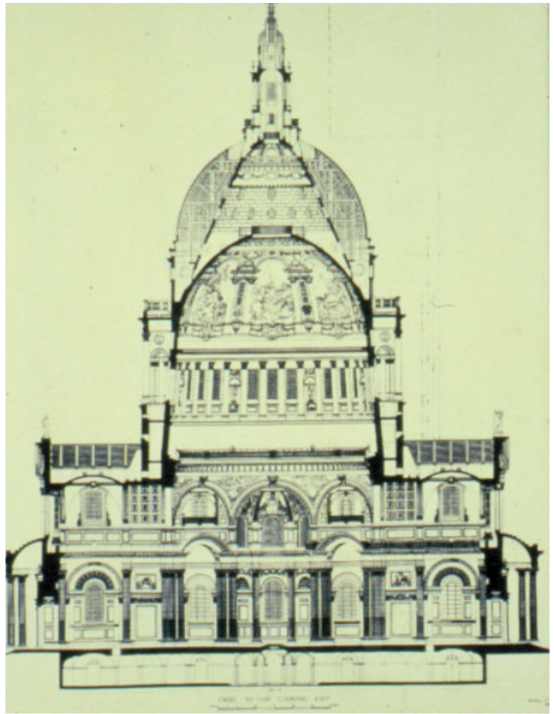
human skull

Shells in Civil Engineering and Architecture

Cupolas and vaults

Pantheon, Rom, 27 b.c.
D = 44m, R/t = 18

TajMahal, India



St. Paul‘s, London, 1506
R/t = 18

Shells in Civil Engineering and Architecture

modern cooling tower
R/t >> 500

concrete, R/t = 520
Kresge Hall, MIT

concrete roof, Heinz Isler

Shells in automotive engineering

pictures:
DaimlerChrysler
BMW
Jaguar

Shells in naval and aerospace engineering

pictures:
Aida
Airbus
Saab
Space Shuttle

Shells in everyday's life

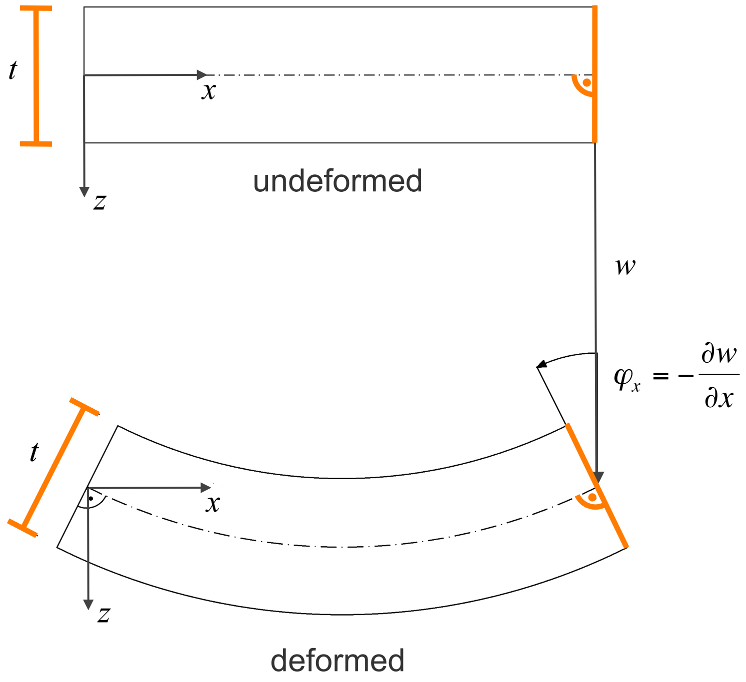
pictures: Braun, CocaCola, Hyve

Shell Theories

Shell models can be classified into:

- Kirchhoff–Love, classical shell theory
- Reissner–Mindlin, shear deformable theory
- Higher order formulations: thickness-deformable, multilayer, multidirector

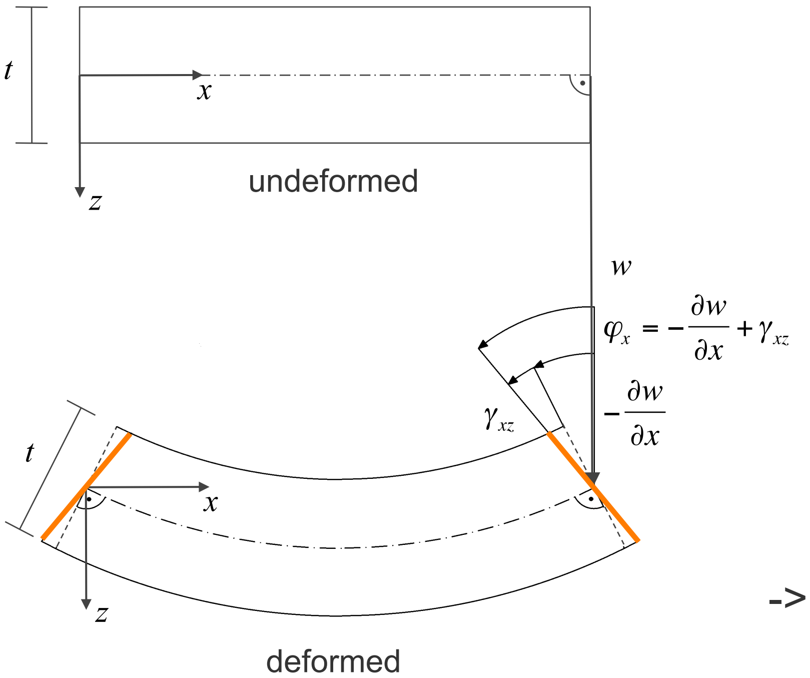
Kirchhoff-Love



Kinematic assumptions:

- Thickness remains unchanged $\varepsilon_{zz} = 0$
- Cross sections remain straight
- Cross sections remain normal

Reissner-Mindlin

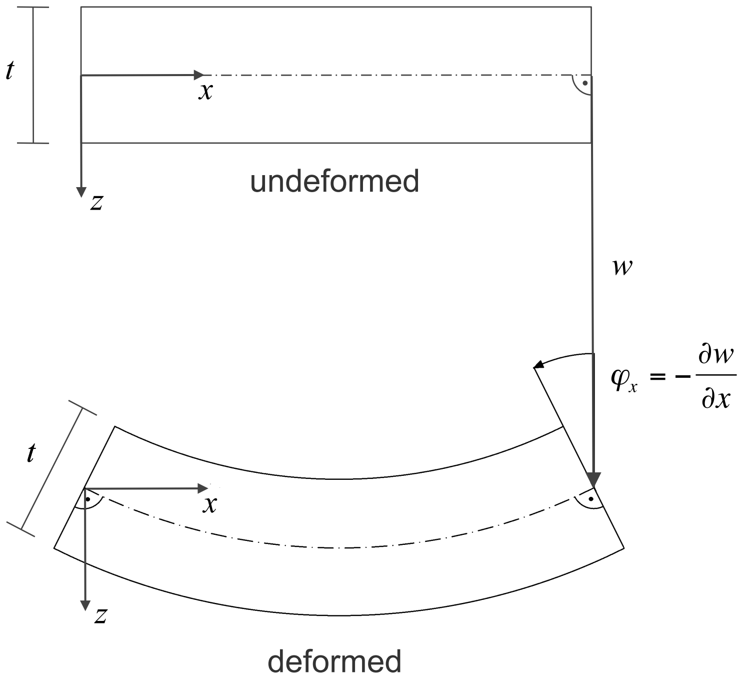


Kinematic assumptions:

- Thickness remains unchanged $\varepsilon_{zz} = 0$
- Cross sections remain straight
- ~~Cross sections remain normal~~

-> transverse shear strain γ_{xz}

Kirchhoff-Love



Kinematic assumptions:

- Thickness remains unchanged $\varepsilon_{zz} = 0$
- Cross sections remain straight
- Cross sections remain normal

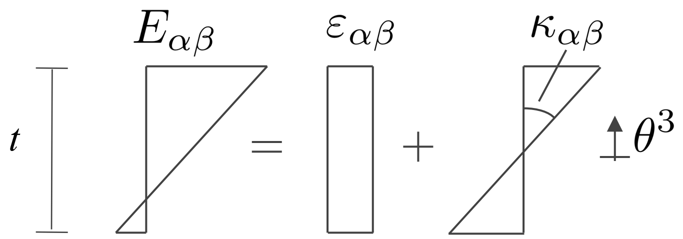
-> transverse shear strain $\gamma_{xz} = 0$

Strains

Kirchhoff assumptions -> no strains in thickness direction $\varepsilon_{i3} = \varepsilon_{3i} = 0$

-> in-plane strains linear through thickness

Separating in-plane strains into constant and symmetrically linear part



- Constant part = membrane strain $\varepsilon_{\alpha\beta}$

- Symmetric linear part = bending strain

  -> $\kappa_{\alpha\beta}$ change in curvature

$$E_{\alpha\beta} = \varepsilon_{\alpha\beta} + \theta^3 \kappa_{\alpha\beta}$$

$$\varepsilon_{\alpha\beta} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} \end{bmatrix} \qquad \kappa_{\alpha\beta} = \begin{bmatrix} \kappa_{11} & \kappa_{12} \\ \kappa_{21} & \kappa_{22} \end{bmatrix}$$

Geometric non-linearity

• Geometrically linear:

- linear relation between strain and deformation
- strain obtained as derivatives of displacements

$$\varepsilon_x = \frac{\partial u_x}{\partial x} \qquad \kappa_x = -\frac{\partial^2 w}{\partial x^2}$$

-> valid only for small deformations

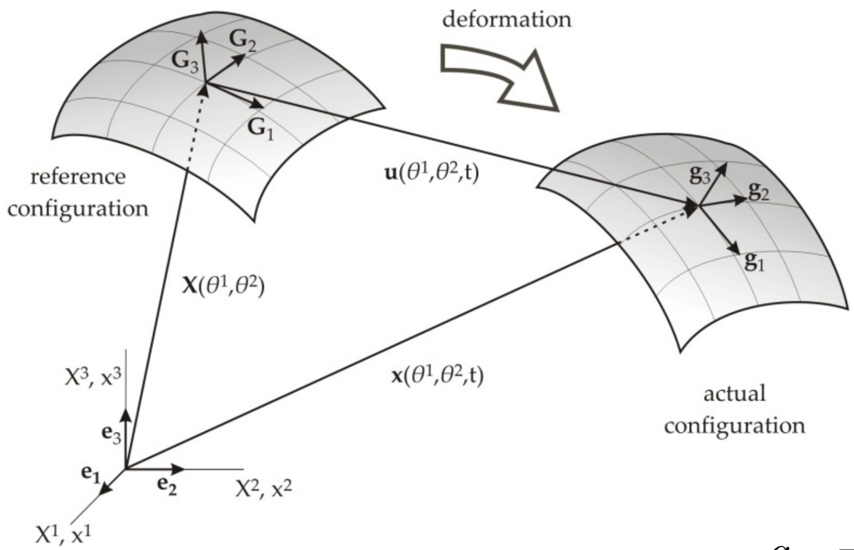
• Geometrically non-linear:

- non-linear relation between strain and deformation
- various non-linear strain measures
- study of geometry in deformed and undeformed configuration

-> valid for small and large deformations

Geometric non-linearity

Study of geometry in deformed/actual and undeformed/reference configuration



$$\mathbf{u} = \mathbf{x} - \mathbf{X}$$

$\mathbf{u}$ - displacement vector

$\mathbf{x}$ - position vector actual configuration

$\mathbf{X}$ - position vector reference configuration

$\mathbf{g}_i$ - local base vectors actual configuration

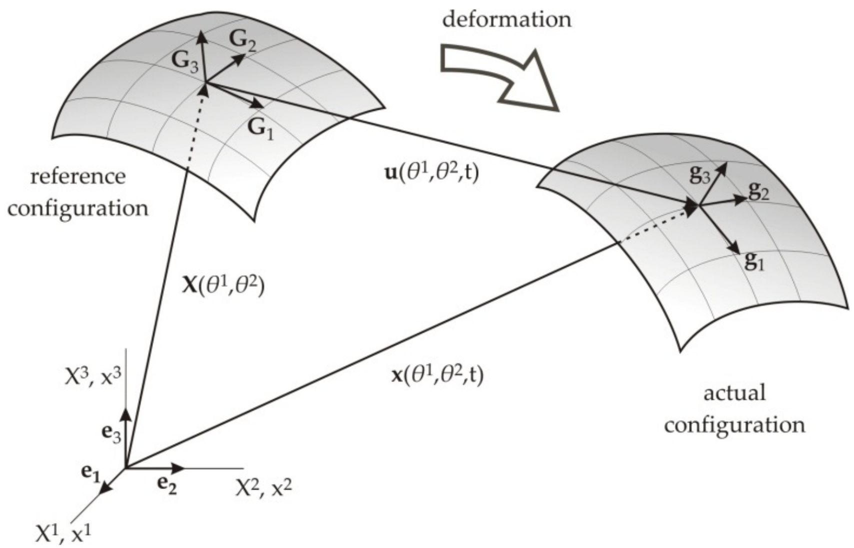
$\mathbf{G}_i$ - local base vectors reference configuration

$g_{ij} = \mathbf{g}_i \cdot \mathbf{g}_j$ - metric tensor actual configuration

$G_{ij} = \mathbf{G}_i \cdot \mathbf{G}_j$ - metric tensor reference configuration

Deformation gradient

Deformation gradient tensor:



$$\mathbf{F} = \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\mathbf{X}}$$

$$\mathrm{d}\mathbf{x} = \mathbf{F} \cdot \mathrm{d}\mathbf{X}$$

$\mathrm{d}\mathbf{x}$ - infinitesimal vector actual configuration

$\mathrm{d}\mathbf{X}$ - infinitesimal vector reference configuration

$$\mathbf{F} = \mathbf{g}_i \otimes \mathbf{G}^i$$

-> The deformation gradient tensor contains all information relative to local strain.

-> However, contains rigid body motions -> no objective strain measure

Strain and stress tensors

Deformation gradient tensor: $\mathbf{F} = \mathbf{g}_i \otimes \mathbf{G}^i$

Euler-Almansi strain tensor: $\mathbf{e} = \frac{1}{2}(\mathbf{I} - \mathbf{F}^{-T}\mathbf{F}^{-1})$

Green-Lagrange strain tensor: $\mathbf{E} = \frac{1}{2}(\mathbf{F}^T\mathbf{F} - \mathbf{I})$ $\mathbf{I}$ - identity tensor

$$E_{ij} = \frac{1}{2}(g_{ij} - G_{ij})$$

g_{ij}, G_{ij} - metric coefficients

Second Piola-Kirchhoff stress tensor: $\mathbf{S} = \mathbf{C} : \mathbf{E}$ $\mathbf{C}$ - material tensor

$$\mathbf{S} = \det\mathbf{F} \cdot \mathbf{F}^{-1} \cdot \boldsymbol{\sigma} \cdot \mathbf{F}^{-T}$$

$\boldsymbol{\sigma}$ - Cauchy stress tensor

Strain and stress tensors

Deformation gradient tensor: $\mathbf{F} = \mathbf{g}_i \otimes \mathbf{G}^i$

Euler-Almansi strain tensor: $\mathbf{e} = \frac{1}{2}(\mathbf{I} - \mathbf{F}^{-T}\mathbf{F}^{-1})$

Green-Lagrange strain tensor: $\mathbf{E} = \frac{1}{2}(\mathbf{F}^T\mathbf{F} - \mathbf{I})$ $\mathbf{I}$ - identity tensor

$$E_{\alpha\beta} = \frac{1}{2}(g_{\alpha\beta} - G_{\alpha\beta})$$

$g_{\alpha\beta}, G_{\alpha\beta}$ - metric coefficients

Second Piola-Kirchhoff stress tensor: $\mathbf{S} = \mathbf{C} : \mathbf{E}$ $\mathbf{C}$ - material tensor

$$\mathbf{S} = \det\mathbf{F} \cdot \mathbf{F}^{-1} \cdot \boldsymbol{\sigma} \cdot \mathbf{F}^{-T}$$

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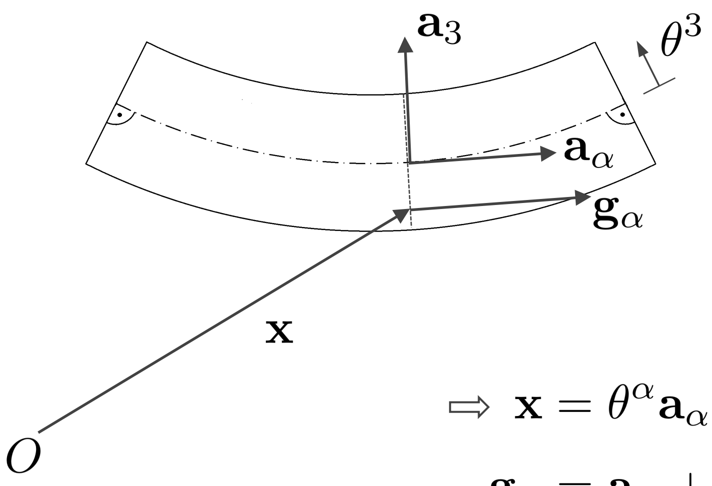
From continuum to shell kinematics

Shell model = surface model
=> relate all quantities of the shell continuum to its middle surface



Shell continuum: $\mathbf{x}$

$$\mathbf{g}_\alpha = \frac{\partial \mathbf{x}}{\partial \theta^\alpha} = \mathbf{x},_\alpha \qquad \alpha = \{1,2\}$$

Shell middle surface: $\mathbf{a}_\alpha = \mathbf{g}_\alpha(\theta^3 = 0)$

$$\mathbf{a}_3 = \frac{\mathbf{a}_1 \times \mathbf{a}_2}{|\mathbf{a}_1 \times \mathbf{a}_2|}$$

$$\Rightarrow \; \mathbf{x} = \theta^\alpha \mathbf{a}_\alpha + \theta^3 \mathbf{a}_3$$

$$\mathbf{g}_\alpha = \mathbf{a}_\alpha + \theta^3 \mathbf{a}_{3,\alpha}$$

Metric tensor: $g_{\alpha\beta} = a_{\alpha\beta} - 2\,\theta^3\, b_{\alpha\beta} + (\theta^3)^2\, \mathbf{a}_{3,\alpha} \cdot \mathbf{a}_{3,\beta}$

neglect quadratic term for linear strain distribution

metric tensor of the midsurface
"1st fundamental form"

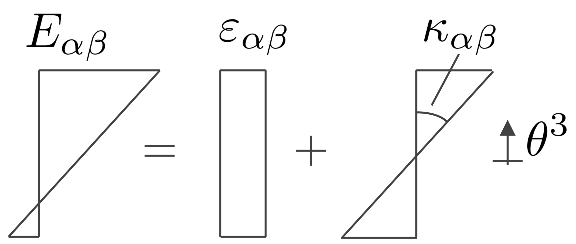
curvature tensor of the midsurface
"2nd fundamental form"

$$b_{\alpha\beta} = \mathbf{a}_{\alpha,\beta} \cdot \mathbf{a}_3$$

Strain tensor: $E_{\alpha\beta} = \frac{1}{2}\boxed{(a_{\alpha\beta} - A_{\alpha\beta})} + \theta^3 \boxed{(B_{\alpha\beta} - b_{\alpha\beta})}$

Strains

Separating in-plane strains into constant and symmetrically linear part



- Constant part = membrane strain $\varepsilon_{\alpha\beta}$
- Symmetric linear part = bending strain

  -> $\kappa_{\alpha\beta}$ change in curvature

$$E_{\alpha\beta} = \varepsilon_{\alpha\beta} + \theta^3 \kappa_{\alpha\beta}$$

$$E_{\alpha\beta} = \frac{1}{2}(a_{\alpha\beta} - A_{\alpha\beta}) + \theta^3 (B_{\alpha\beta} - b_{\alpha\beta})$$

$$\varepsilon_{\alpha\beta} = \frac{1}{2}(a_{\alpha\beta} - A_{\alpha\beta})$$ -> Membrane strain from metric tensors

$$\kappa_{\alpha\beta} = B_{\alpha\beta} - b_{\alpha\beta}$$ -> Bending strain from curvature tensors

Variational Formulation

Principle of virtual work

Variation δu_r

$$\delta W = \frac{\partial W}{\partial u_r}\delta u_r = 0$$

Linearization

$$\frac{\partial W}{\partial u_r} + \frac{\partial^2 W}{\partial u_r \partial u_s}\Delta u_s = 0$$

$$\downarrow \qquad \downarrow$$

$$\mathbf{R} - \mathbf{K}\Delta\mathbf{u} = 0$$

R - residual force vector
K - stiffness matrix
Δ**u** - displacement vector

Variational Formulation

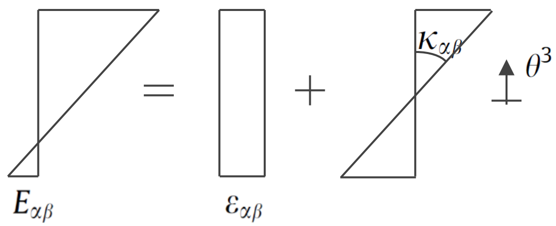
Internal virtual work $\delta W_{int} = -\int_{\Omega} (\mathbf{S} : \delta \mathbf{E}) \, d\Omega$

S - Piola-Kirchhoff 2 stress tensor
E - Green-Lagrange strain tensor

=> Separating strain into membrane and bending action

$\boldsymbol{\varepsilon}$ - membrane strain
$\mathbf{n}$ - normal forces $\quad \mathbf{n} = t\,\mathbf{C} : \boldsymbol{\varepsilon}$
$\boldsymbol{\kappa}$ - change of curvature
$\mathbf{m}$ - bending moments $\quad \mathbf{m} = \frac{t^3}{12}\mathbf{C} : \boldsymbol{\kappa}$

$$E_{\alpha\beta}(\theta^3) = \varepsilon_{\alpha\beta} + \theta^3 \kappa_{\alpha\beta}$$



$$\delta W_{int} = -\int_A (\mathbf{n} : \delta\boldsymbol{\varepsilon} + \mathbf{m} : \delta\boldsymbol{\kappa}) \, dA$$

$$F_r^{int} = -\int_A \left(\mathbf{n} : \frac{\partial \boldsymbol{\varepsilon}}{\partial u_r} + \mathbf{m} : \frac{\partial \boldsymbol{\kappa}}{\partial u_r} \right) dA$$

$$K_{rs}^{int} = \int_A \left(\frac{\partial \mathbf{n}}{\partial u_s} : \frac{\partial \boldsymbol{\varepsilon}}{\partial u_r} + \mathbf{n} : \frac{\partial^2 \boldsymbol{\varepsilon}}{\partial u_r \partial u_s} + \frac{\partial \mathbf{m}}{\partial u_s} : \frac{\partial \boldsymbol{\kappa}}{\partial u_r} + \mathbf{m} : \frac{\partial^2 \boldsymbol{\kappa}}{\partial u_r \partial u_s} \right) dA$$

=> Problem for FE: curvature -> 2nd derivatives -> C^1 continuity between elements

Isogeometric Analysis for KL Shell formulation

NURBS-based IGA: -> Exact geometry in analysis model

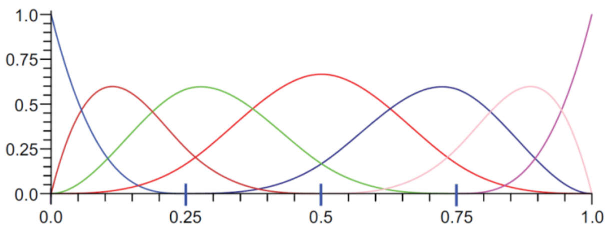
⇒ C^1 and higher continuity between elements provided by NURBS

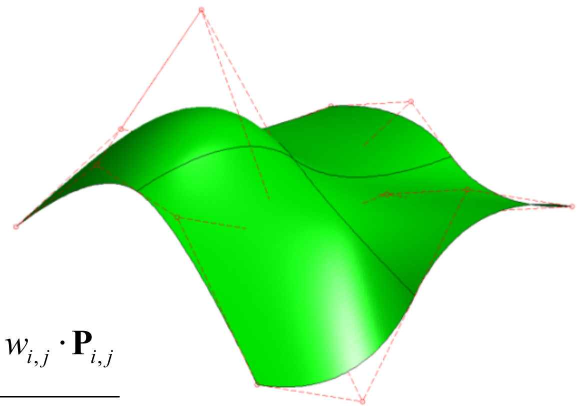
⇒ exact evaluation of geometric properties including curvatures

⇒ straightforward implementation of kinematic formulation; no further assumptions

⇒ no rotational degrees of freedom

⇒ no shear locking !



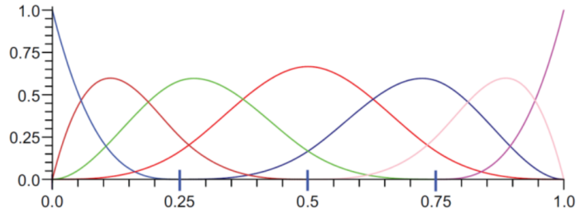


$$\mathbf{S}(u,v) = \frac{\sum_{j=1}^{m}\sum_{i=1}^{n} N_{i,p}(u)\cdot M_{j,q}(v)\cdot w_{i,j}\cdot \mathbf{P}_{i,j}}{\sum_{j=1}^{m}\sum_{i=1}^{n} N_{i,p}(u)\cdot M_{j,q}(v)\cdot w_{i,j}}$$

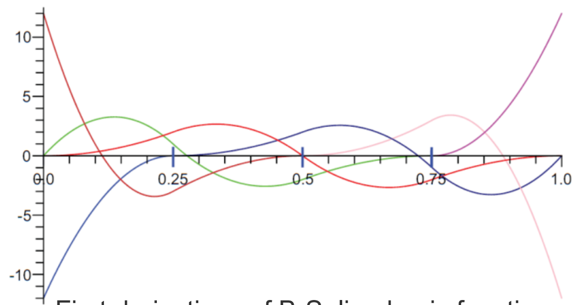
Rotational boundary conditions

Open knot vector: U = {0, 0, 0, 0, 0.25, 0.5, 0.75, 1, 1, 1, 1}

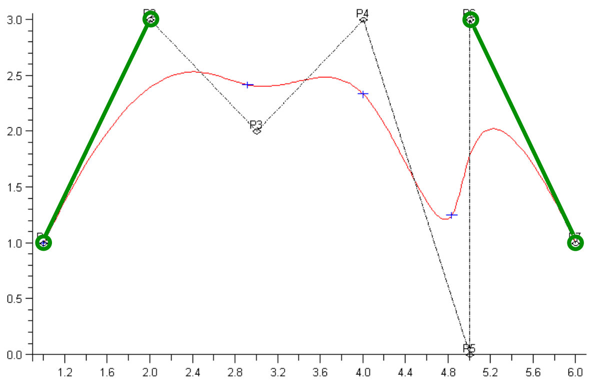
p+1 p+1



B-Spline basis functions



First derivatives of B-Spline basis functions



-> first and last control point points are interpolated

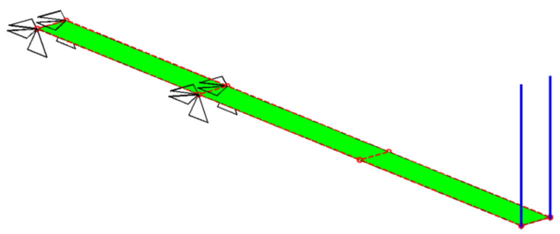
-> second and next to last control point define the curve's tangents at the ends

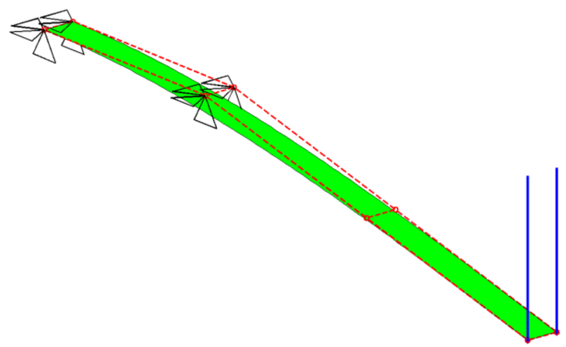
Rotational boundary conditions

Needed for: clamped support, symmetry conditions

-> Tangent at the boundary is determined by 1st and 2nd row of control points

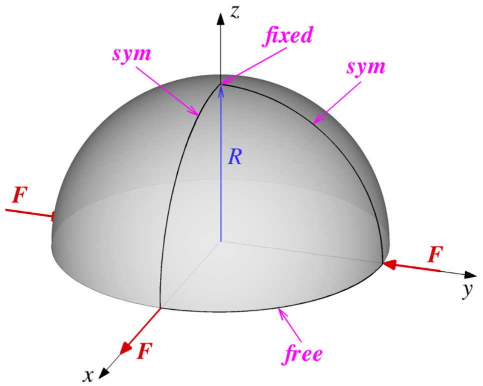
Example: cantilever plate



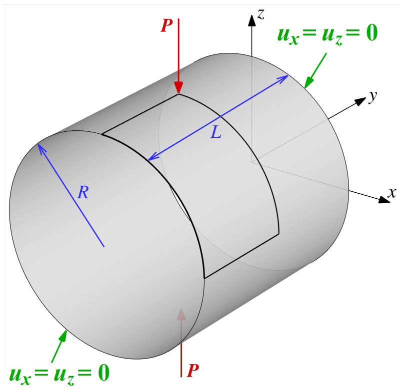


Benchmarking

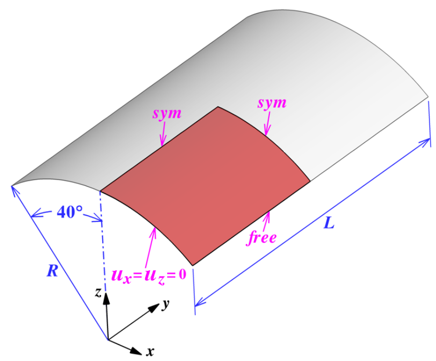
Shell obstacle course



pinched hemisphere
(1/4)



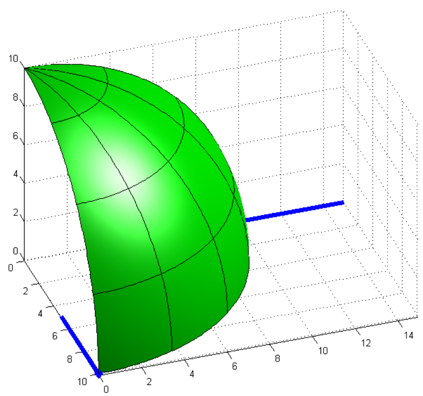
pinched cylinder
(1/8)



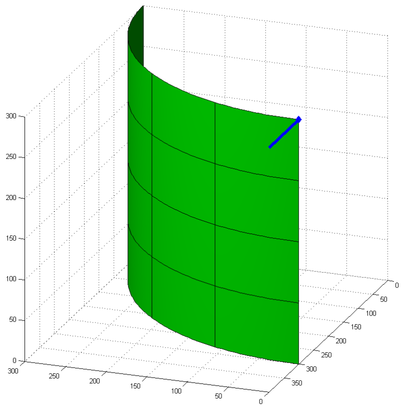
Scordelis-Lo roof
(1/4)

Benchmarking

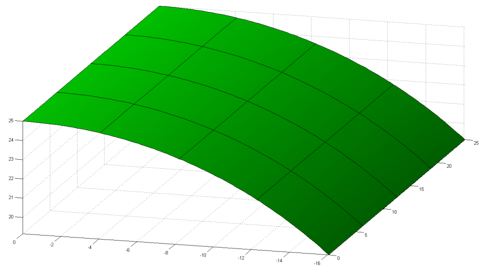
Shell obstacle course



pinched hemisphere
(1/4)

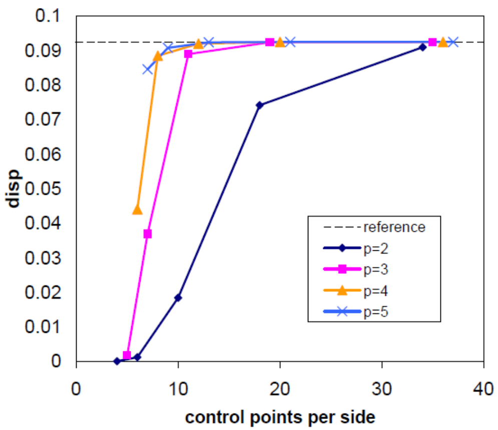


pinched cylinder
(1/8)

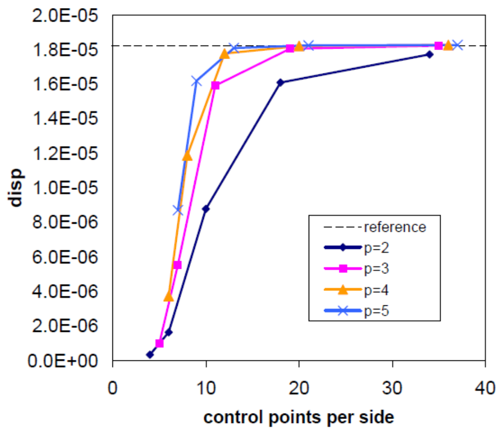


Scordelis-Lo roof
(1/4)

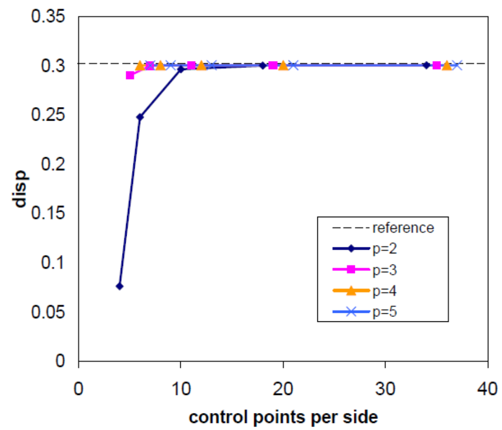
Shell obstacle course – convergence charts



Hemishperical shell



Pinched cylinder



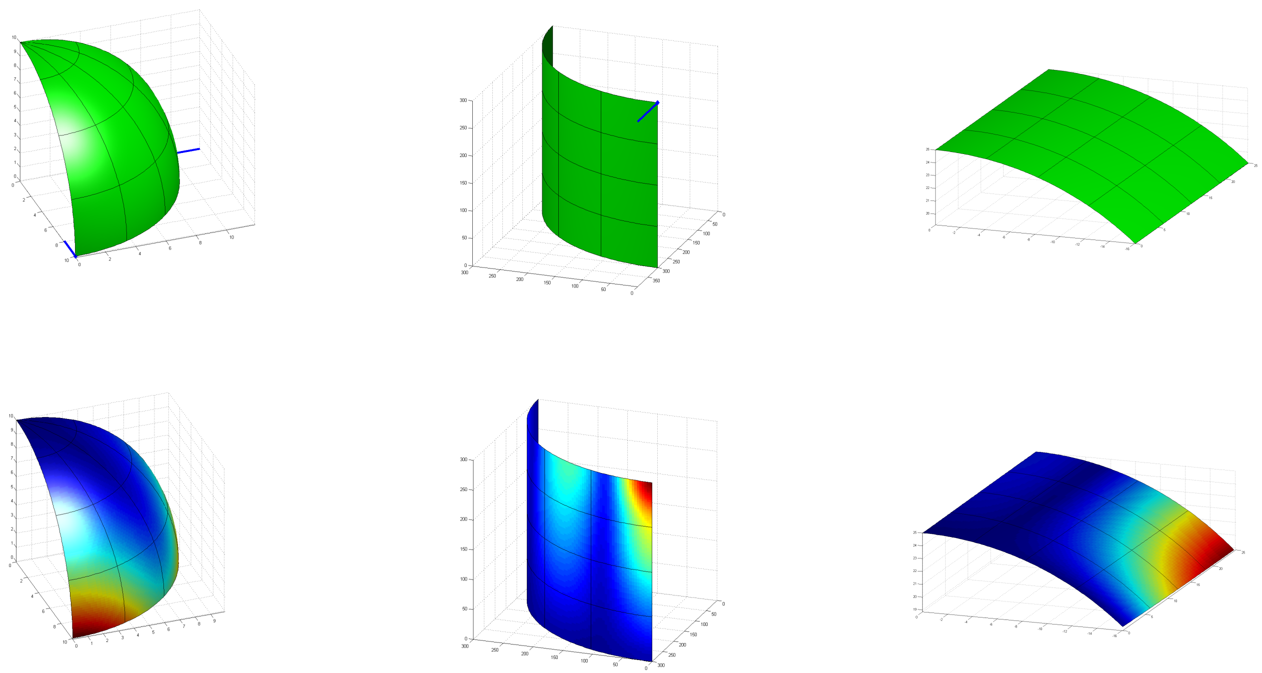
Scordelis-Lo roof

Benchmarking

Shell obstacle course

setup

deformations

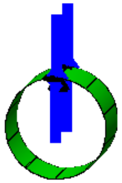


pinched hemisphere | pinched cylinder | Scordelis-Lo roof

Large deformations and rotations

Benchmark: Straight plate bent to a circle

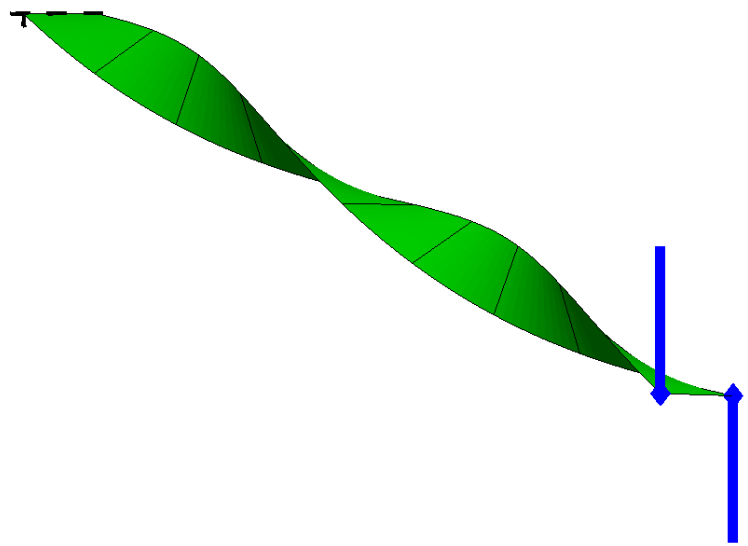
Moment $M = \frac{2\pi EI}{L}$ modeled by a pair of forces perpendicular to the geometry



Large Deformations / Large Rotations

Example 2: Cantilever beam subject to constant twisting moment.

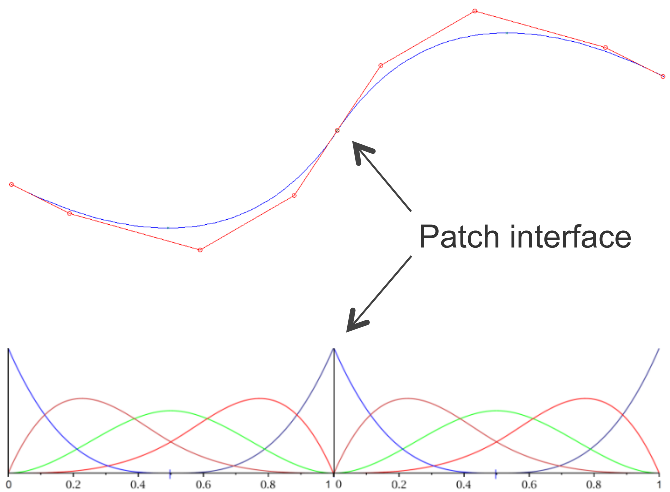
Moment $M_t = \theta \frac{GI_t}{L}$ modeled by a pair of forces perpendicular to the geometry



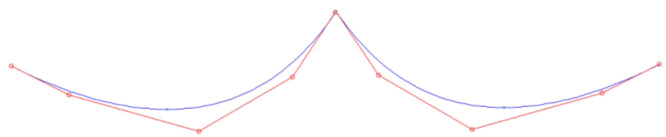
Multiple Patches

NURBS patches are C^0 on the boundary

Smooth Patches:



Non-smooth Patches:



Multiple Patches

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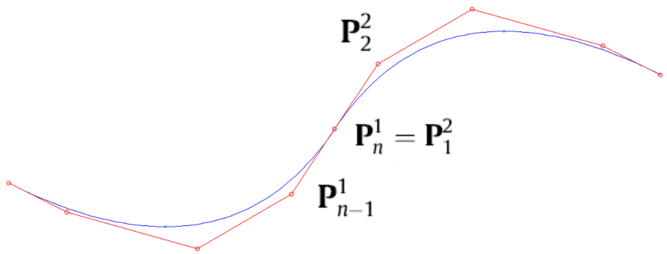
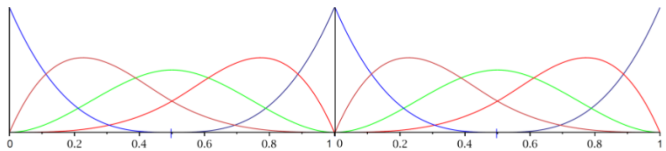
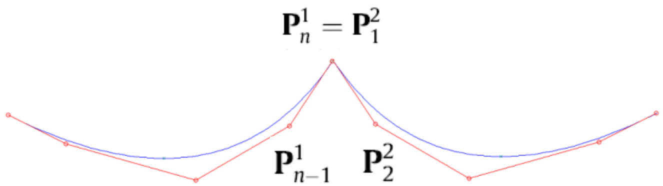


Constraint equation:

$$\mathbf{P}_2^2 = (1 + c)\,\mathbf{P}_n^1 - c\,\mathbf{P}_{n-1}^1$$



Non-smooth Patches:



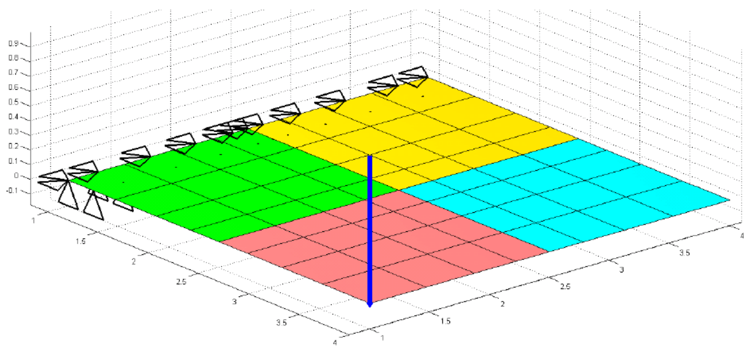
Constraint equation:

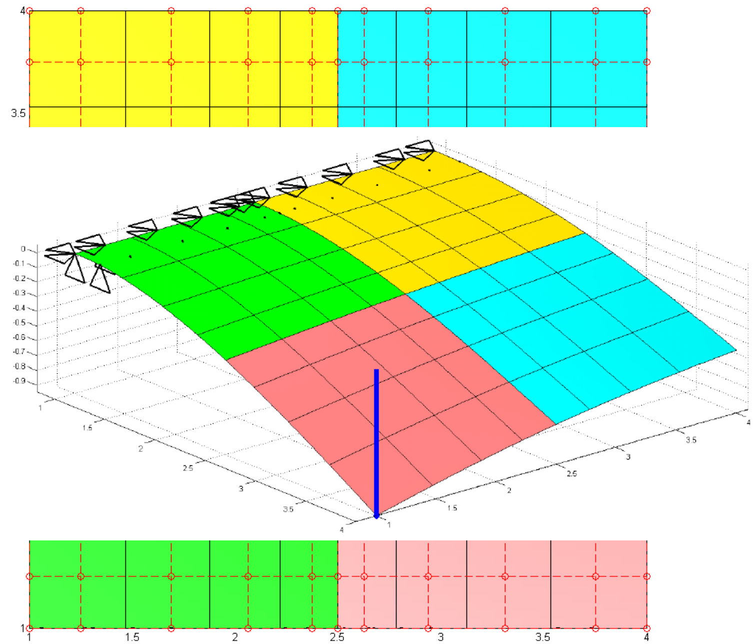
$$\alpha = \cos^{-1}\left(\frac{\left(\mathbf{P}_n^1 - \mathbf{P}_{n-1}^1\right)\cdot\left(\mathbf{P}_2^2 - \mathbf{P}_n^1\right)}{\left|\mathbf{P}_n^1 - \mathbf{P}_{n-1}^1\right|\cdot\left|\mathbf{P}_2^2 - \mathbf{P}_n^1\right|}\right)$$

Smooth, C^1-continuous, Patches

Coupling of control points across patch boundaries

-> continuity constraint is fulfilled exactly

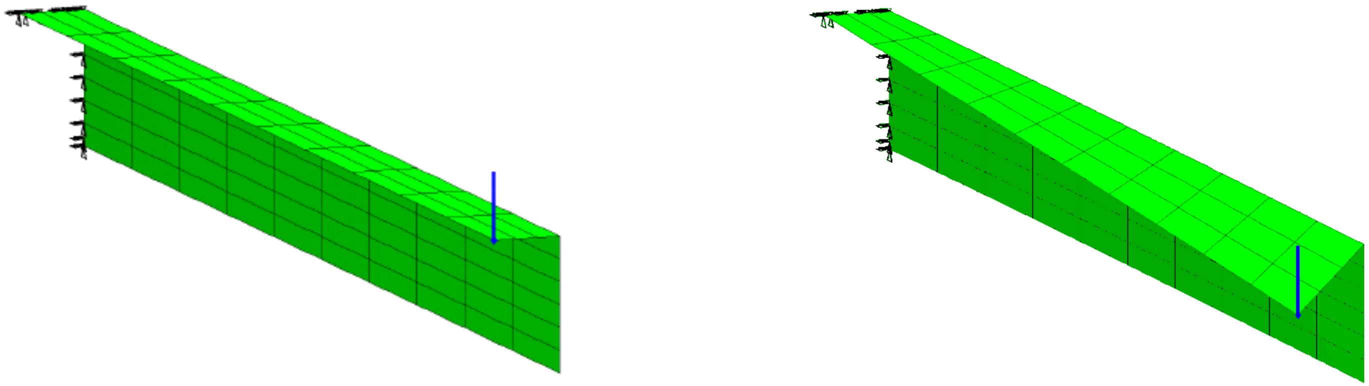




Patches forming a kink

-> Maintain angle between patches. Not possible by direct coupling of control points!

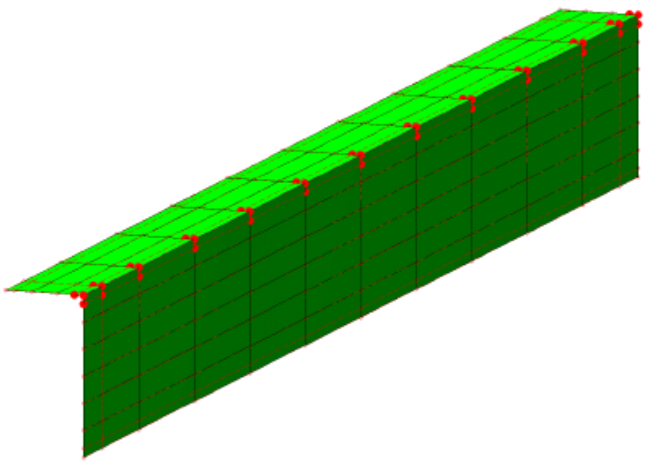
C^0 coupling: no transfer of bending moments => Connection acts as a hinge



L-profile cantilever

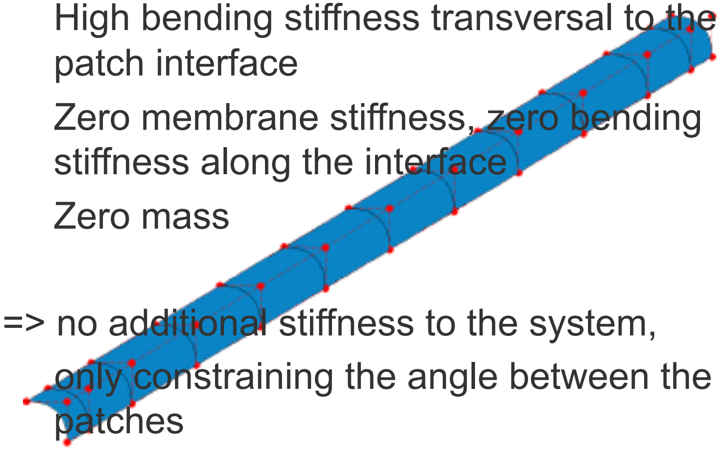
Bending Strips

Couple control points defining the angle between patches by a stiff bending strip



- High bending stiffness transversal to the patch interface
- Zero membrane stiffness, zero bending stiffness along the interface
- Zero mass

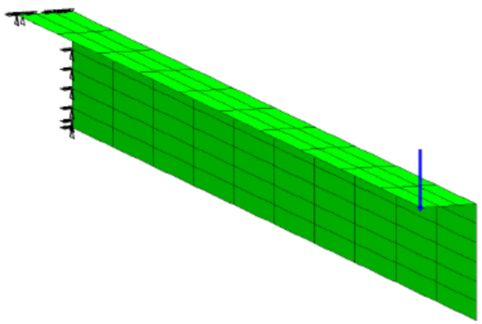
=> no additional stiffness to the system, only constraining the angle between the patches



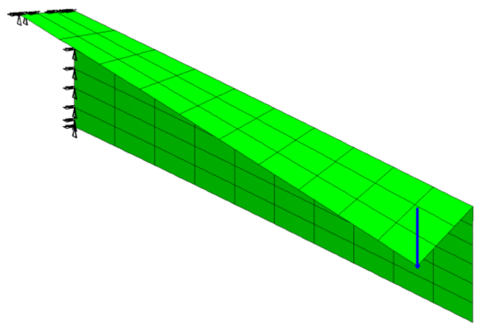
L- profile beam

bending strip

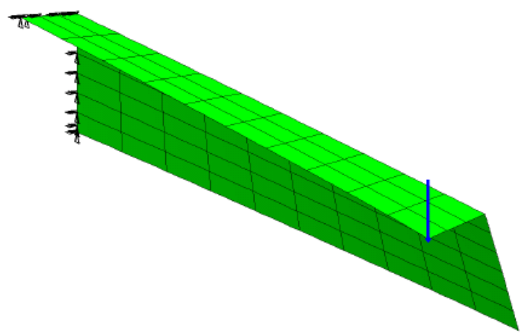
Bending Strips



L-profile cantilever



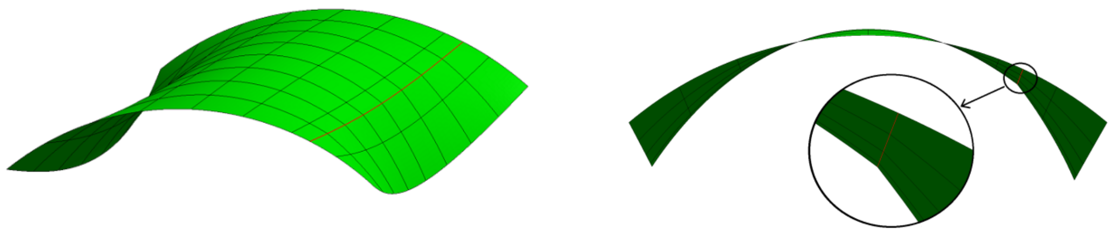
solution without bending strip



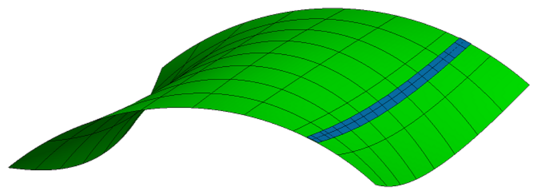
solution with bending strip

Bending Strips for smooth patches

Scordelis-Lo roof modeled by two patches

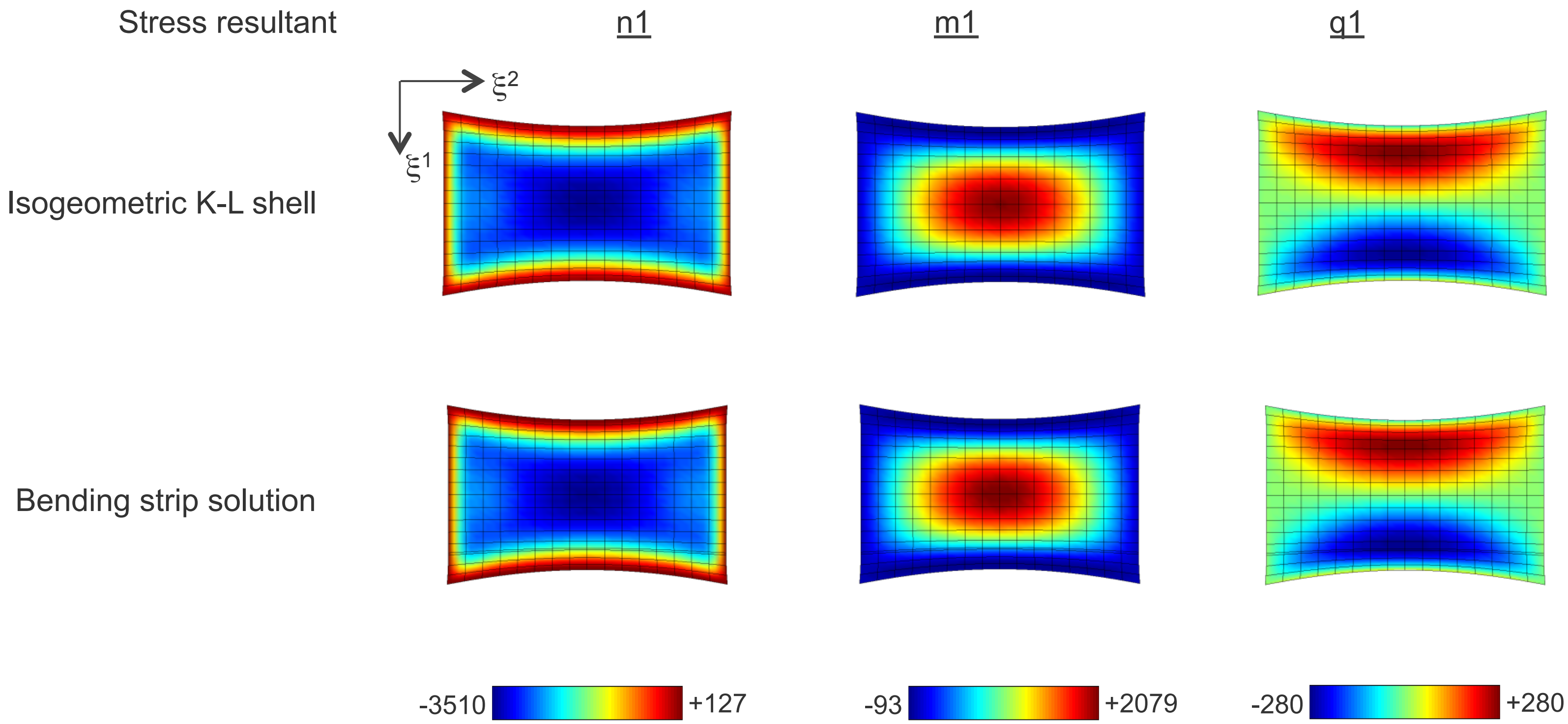


C^0 connection => kink



Correct solution with bending strip

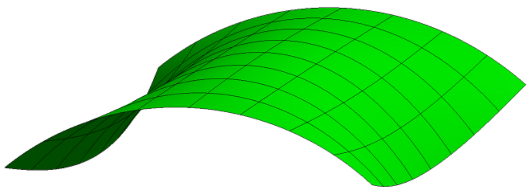
Scordelis-Lo roof – stress resultant plots



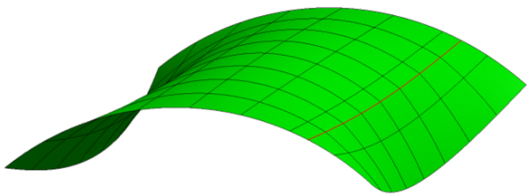
Bending Strips

-> Applicable to both smooth patches and patches with kink

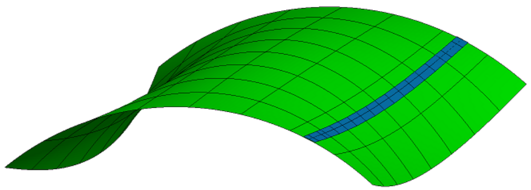
Scordelis-Lo roof – deformation plots



single patch



two patches => kink

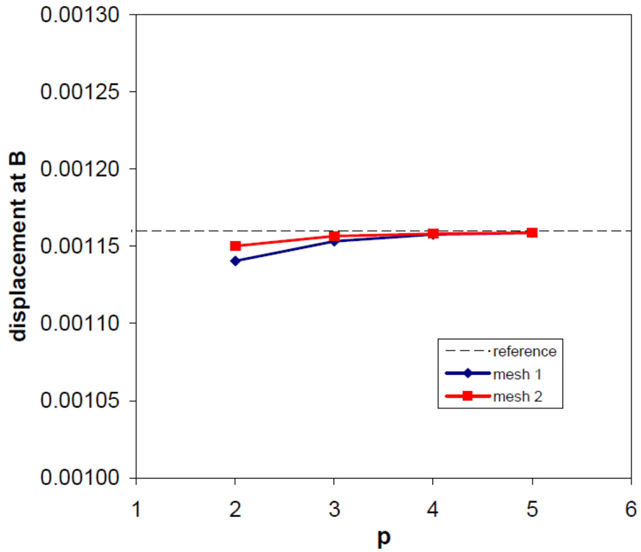


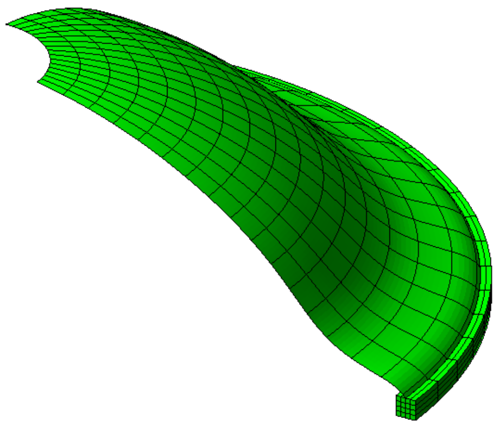
kink repaired by bending strip

=> simple and efficient method to treat arbitrary multipatch structures

Bending Strips for coupling of Shells and Solids

Shell with stiffener (Rank et al.)





Shell modeled by shell elements

Stiffener modeled by solid elements

coupled by a bending strip

Outline

- Introduction
- Kirchhoff-Love shell theory
- Isogeometric shell analysis
- Integration into CAD
- Application to FSI simulations of wind turbine blades
- Isogeometric shape optimization

CAD descriptions are surface-based

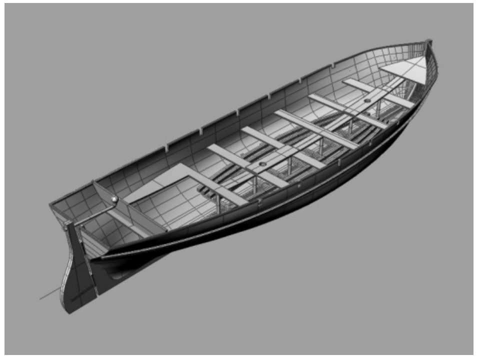
- No volumetric NURBS description in CAD -> creating volumetric model from surface model necessary
- For thin-walled structures: shell analysis on the surface model
- Rotation-free shell => surface model from CAD program can be used for analysis without modification

<u>Rhinoceros</u>

NURBS-based CAD software

for industrial, mechanical, marine, architectural design, etc.

allows for self-written plug-ins -> <u>plug-in for isogeometric analysis</u>



sail cutter
www.viribusunitis.ca

microcupyacht
www.flexicad.com

Opel Astra
www.rhino3d.com

Rhinoceros

NURBS-based CAD software

for industrial, mechanical, marine, architectural design, etc.

Plug-in for isogeometric analysis

Robert Schmidt
Michael Breitenberger

IGA_movie1.mp4 **IGA_movie2.mp4**

Application: FSI simulation of a Wind Turbine Blade

Cooperation project with Y. Bazilevs, M.-C. Hsu, University of California, San Diego

5MW offshore wind turbines:

- Blade radius: 63 m
- Blade material: fibreglass

  E_1=43.3GP E_2=12.7GP G_{12}=4.5GP nu_{12}=0.29
- Speed of rotation: 8-13 RPM
- Wind speed: 8~12 m/s
- Reynolds number: Several hundred million

Fluid and Mesh Motion

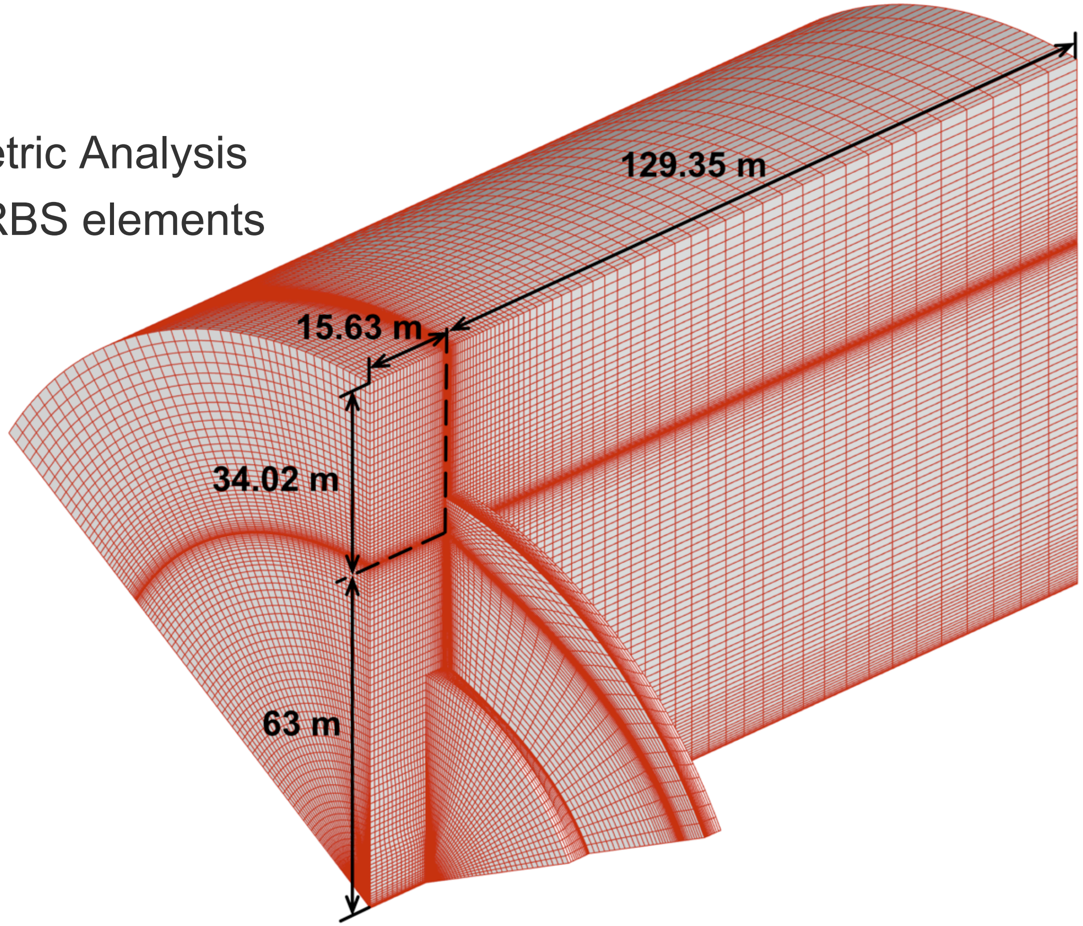
- Fluid:
  - incompressible Newtonian fluid in the ALE description
  - turbulence modeling: residual based variational multiscale method (RBVMS)
  - NURBS as basis functions

- Mesh Motion:
  - divided into rotation (exactly) and blade deflection (linear elasticity)

- Kinematic (strong) and traction (weak) compatibility at the fluid-solid boundary

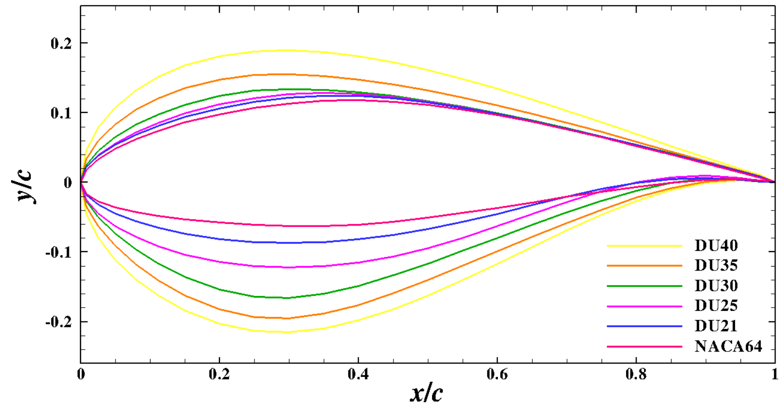
Problem Description

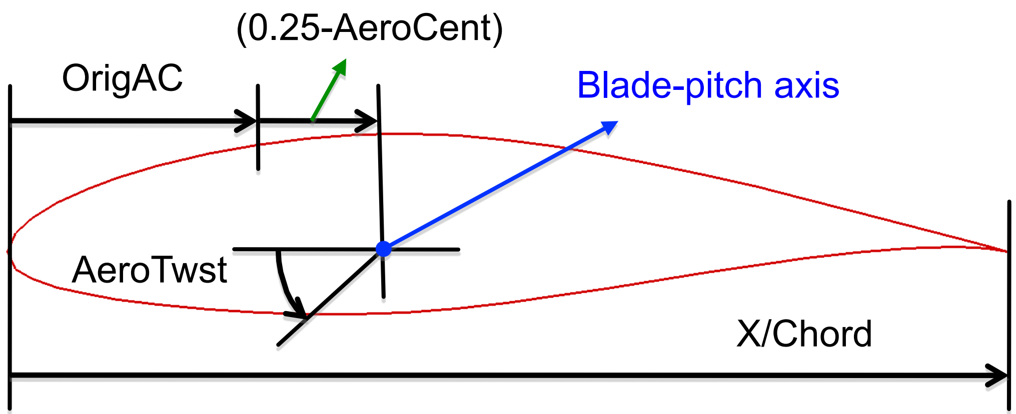
- Full scale simulation
- NURBS-based Isogeometric Analysis
- 1,111,968 quadratic NURBS elements
- 60 processors
- Symmetry condition

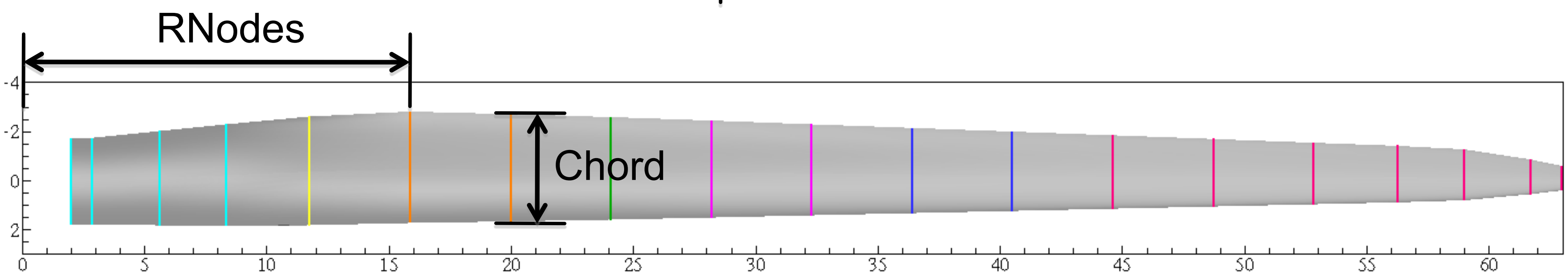


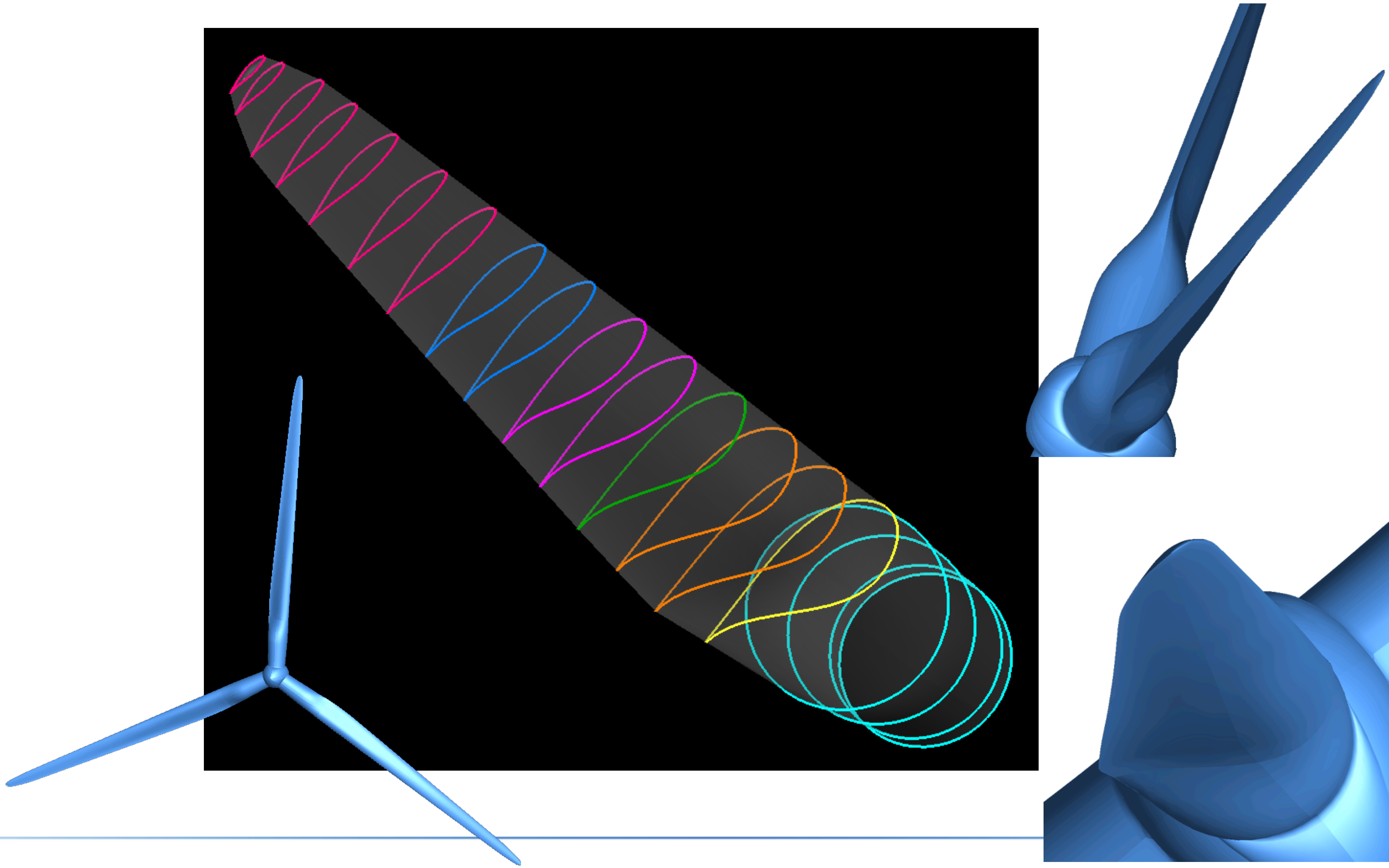
Blade Design: Airfoils

RNodes (m)	AeroTwst (°)	Chord (m)	AeroCent (-)	OrigAC (-)	Airfoil
2.0000	0.000	3.542	0.2500	0.50	Cylinder
2.8667	0.000	3.542	0.2500	0.50	Cylinder
5.6000	0.000	3.854	0.2218	0.44	Cylinder
8.3333	0.000	4.167	0.1883	0.38	Cylinder
11.7500	13.308	4.557	0.1465	0.30	DU40
15.8500	11.480	4.652	0.1250	0.25	DU35
19.9500	10.162	4.458	0.1250	0.25	DU35
24.0500	9.011	4.249	0.1250	0.25	DU30
28.1500	7.795	4.007	0.1250	0.25	DU25
32.2500	6.544	3.748	0.1250	0.25	DU25
36.3500	5.361	3.502	0.1250	0.25	DU21
40.4500	4.188	3.256	0.1250	0.25	DU21
44.5500	3.125	3.010	0.1250	0.25	NACA64
48.6500	2.319	2.764	0.1250	0.25	NACA64
52.7500	1.526	2.518	0.1250	0.25	NACA64
56.1667	0.863	2.313	0.1250	0.25	NACA64
58.9000	0.370	2.086	0.1250	0.25	NACA64
61.6333	0.106	1.419	0.1250	0.25	NACA64
63.0000	0.000	1.000	0.1250	0.25	NACA64

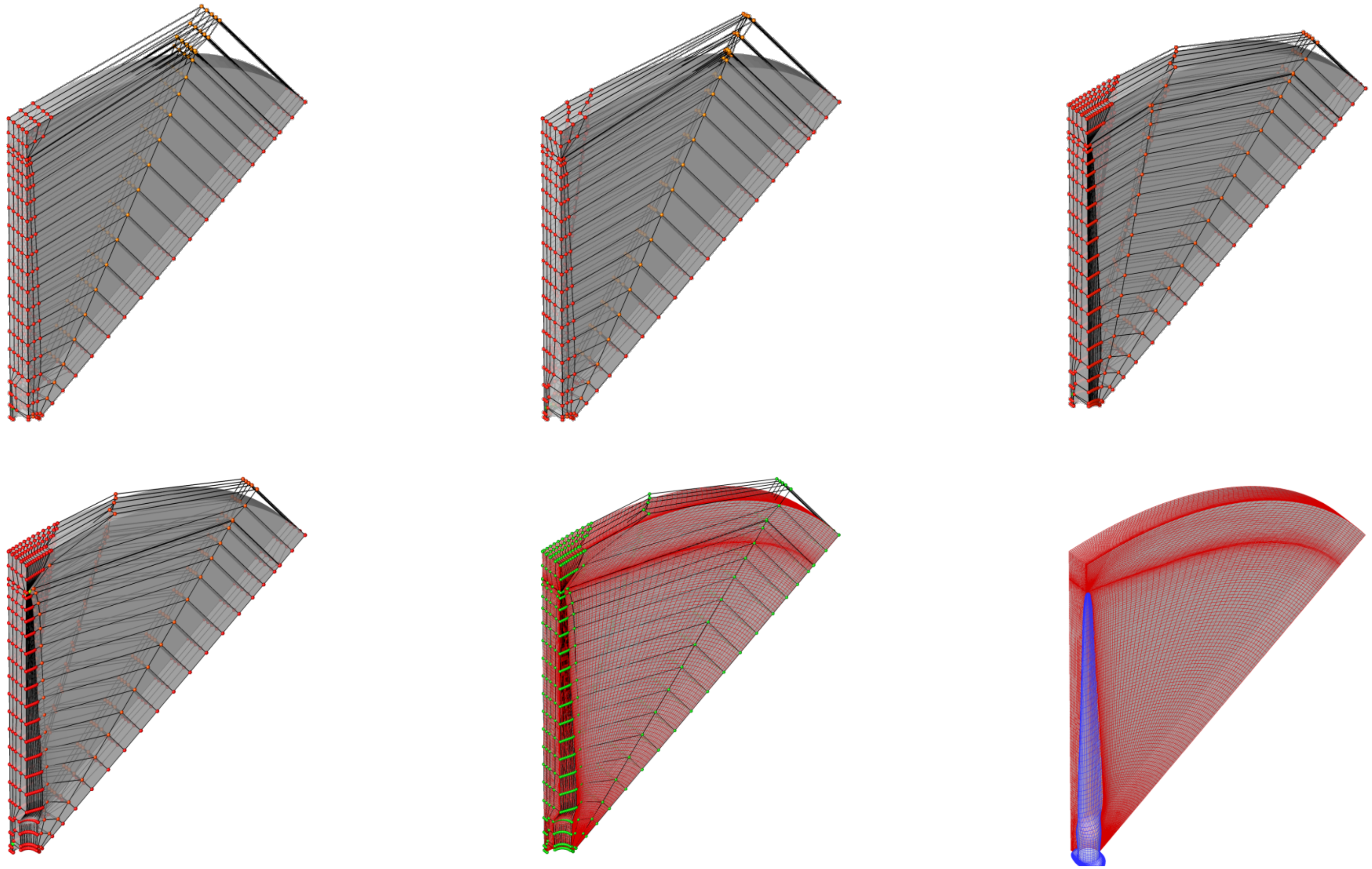




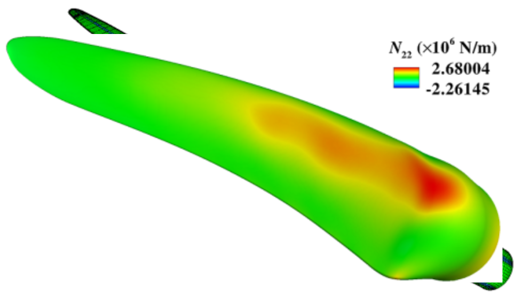




Blade Design: NURBS

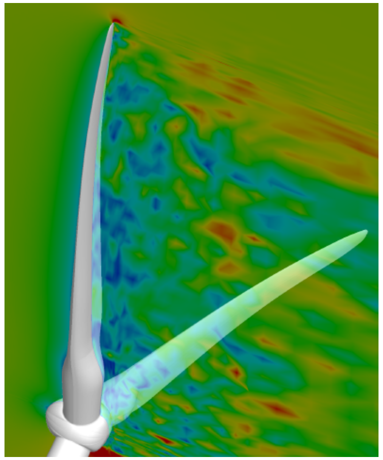


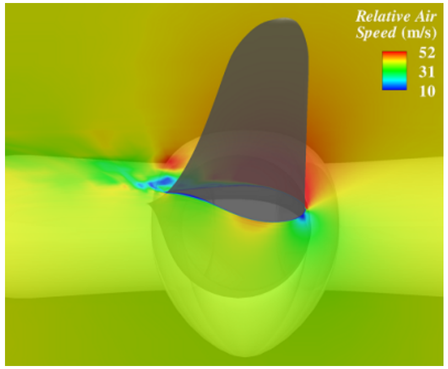
3D FSI Simulation



Wind turbine blade

Normal force





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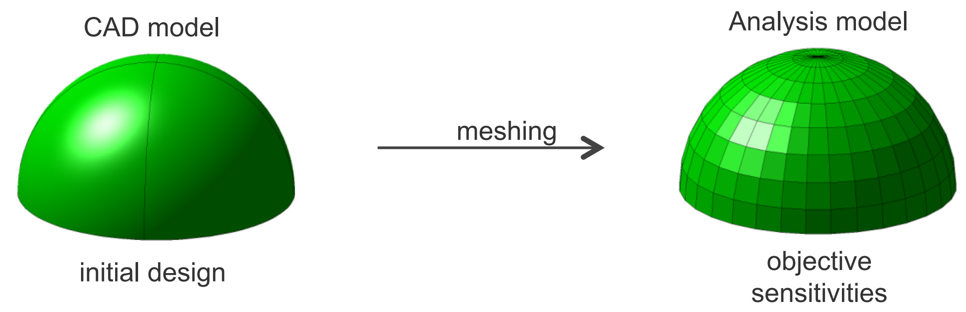
Shape Optimization

- Optimization Problem:

objective function:	$f(\mathbf{s}) \rightarrow \min.$	e.g.: $0.5 \cdot \int \boldsymbol{\sigma\varepsilon} \mathrm{dV} \rightarrow \min.$
equality constraints:	$\mathbf{h}(\mathbf{s}) = \mathbf{0}$	e.g. mass: $m = m_{\mathrm{def}}$
inequality constraints:	$\mathbf{g}(\mathbf{s}) \leq 0$	e.g. stress: $\sigma \leq \sigma_{\max}$
design variables:	$\mathrm{s}_i \quad i = 1,...,\mathrm{n}$	shape variables e.g. $l,\ r \quad x_i, y_i, z_i$

- Shape Optimization with FE -> two different geometry descriptions



- Design Parametrization:
  - CAD-based: CAD parameters
  - FE-based: FE nodes

Shape Optimization

- Optimization Problem:

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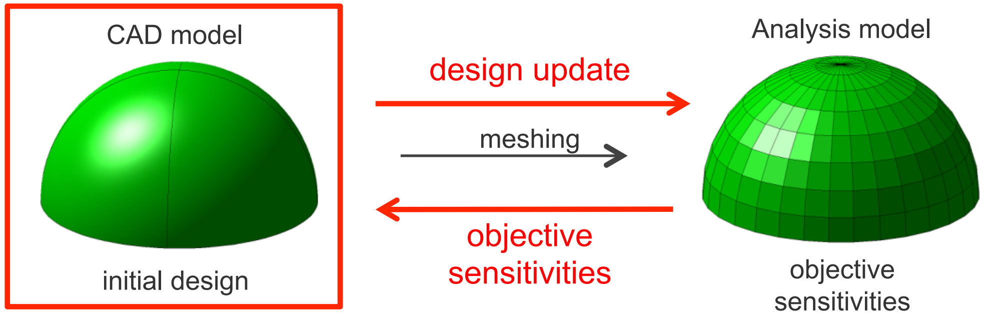

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Shape Optimization

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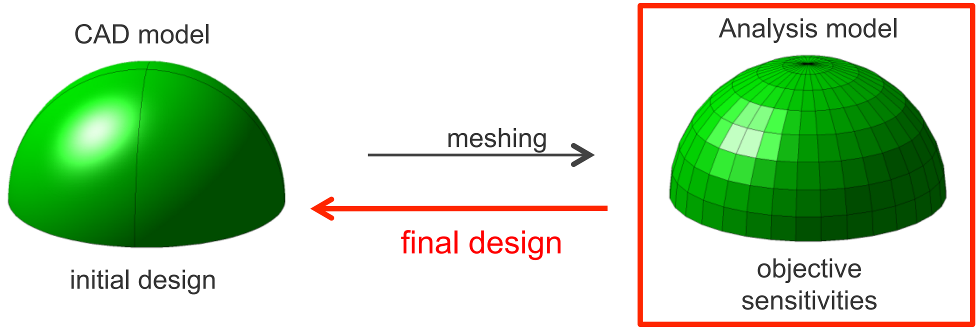
- Shape Optimization with FE -> two different geometry descriptions



- Design Parametrization:
  - CAD-based: CAD parameters
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Shape Optimization

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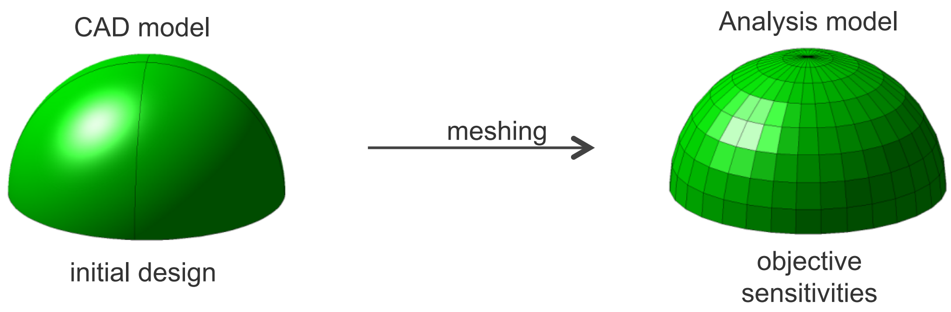
design variables:	$s_i \quad i = 1,...,n$	shape variables e.g. $l,\ r \quad x_i, y_i, z_i$

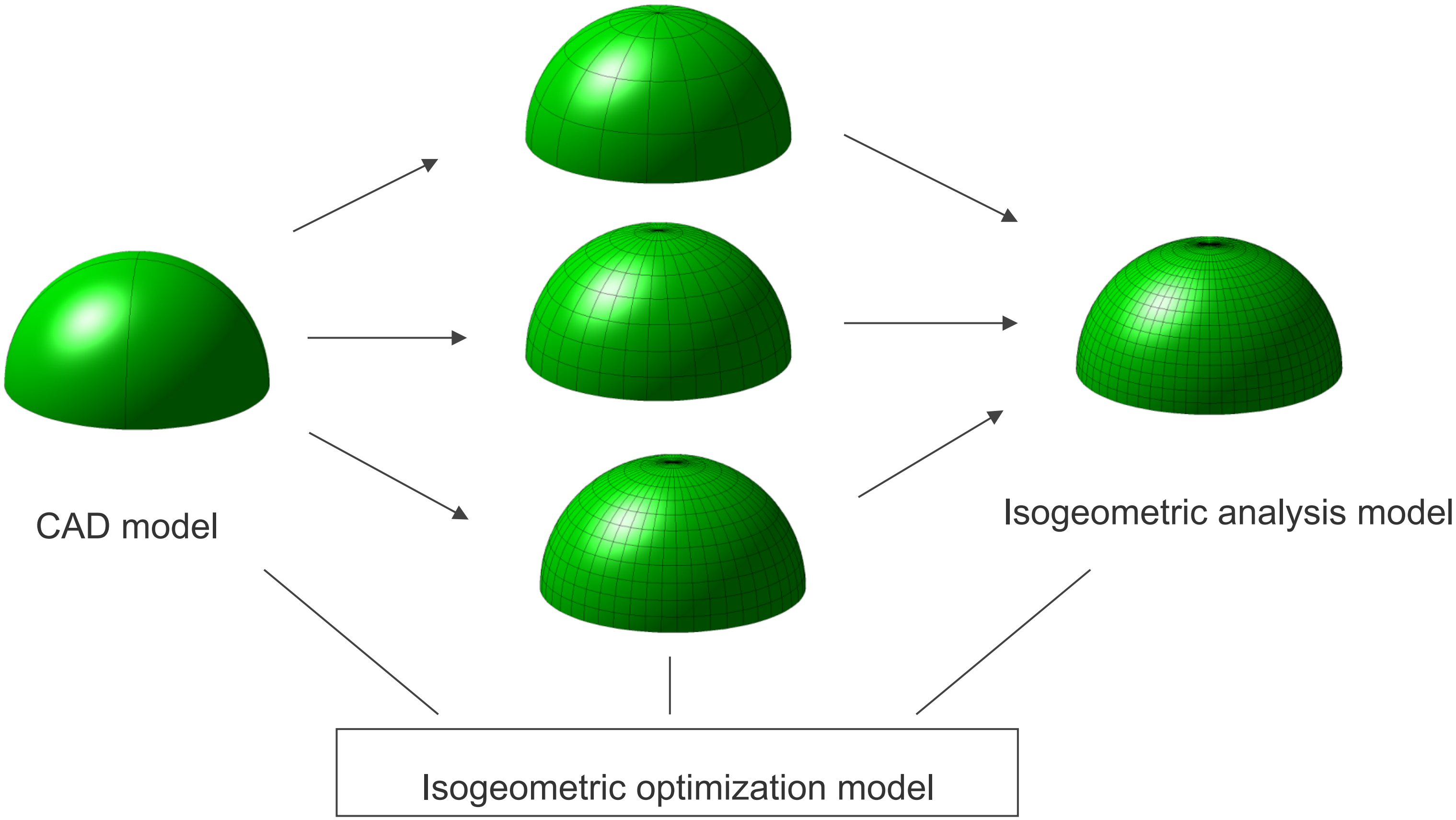
- Shape Optimization with FE -> two different geometry descriptions



- Design Parametrization:
  - CAD-based: CAD parameters
  - FE-based: FE nodes
  - *Isogeometric: Control Points*

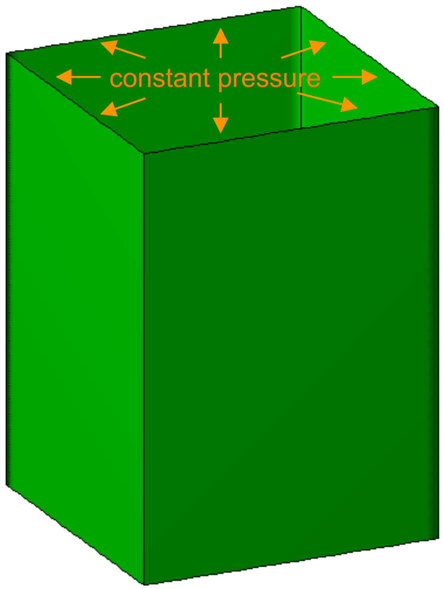
Isogeometric Shape Optimization

Design variables: Control points



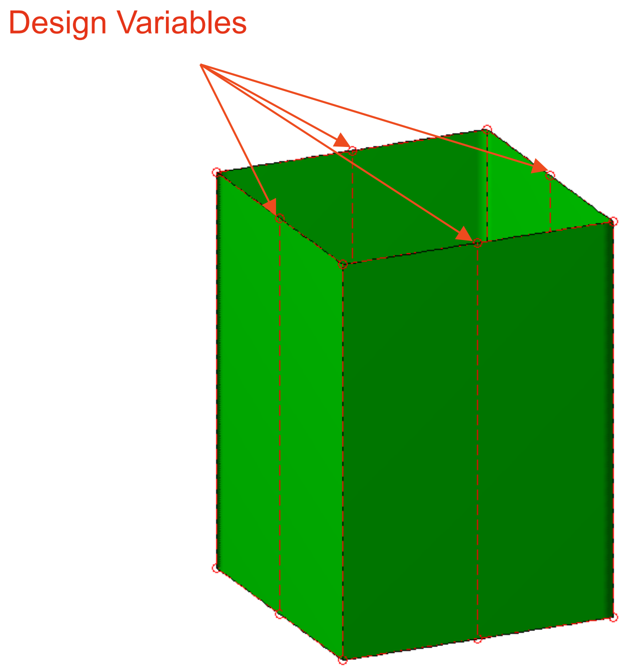
Isogeometric Shape Optimization

Example: Tube under constant internal pressure



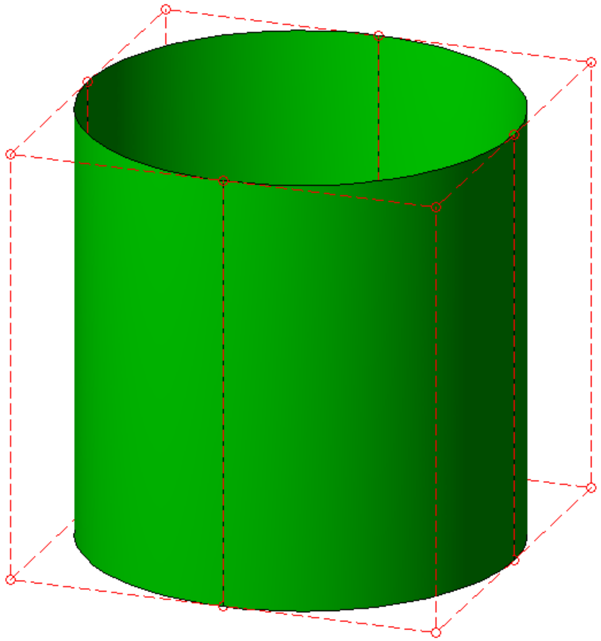
Find optimal shape of the section in order to maximize the stiffness

Example: Tube under constant internal pressure



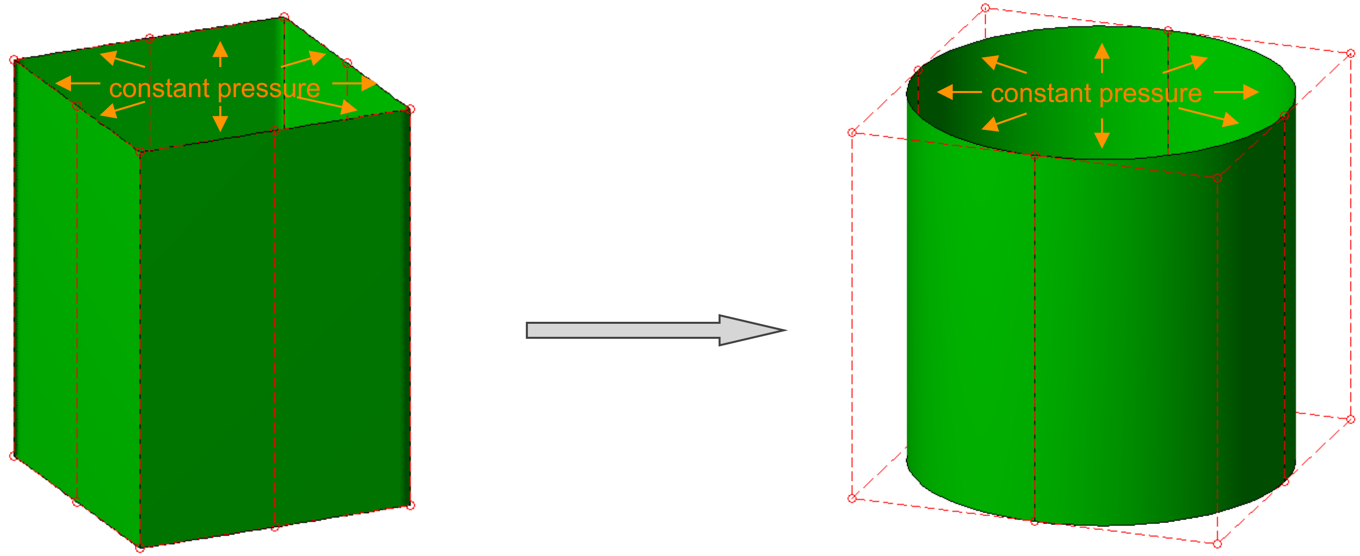
12 design variables:
x, y, w
of the four edge control points

Animation: Optimization steps



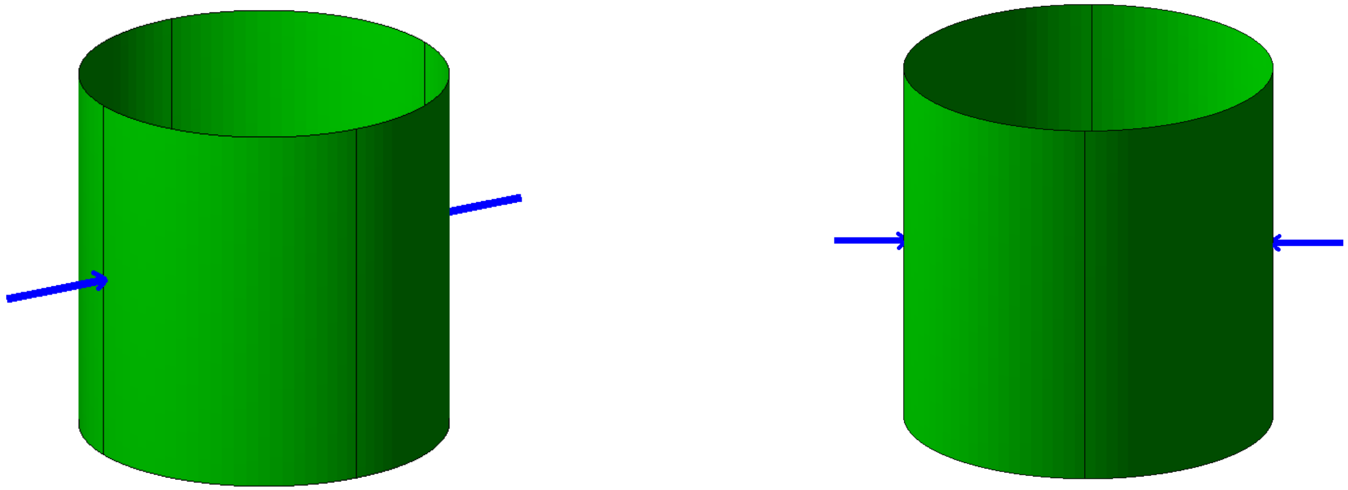
Example: Tube under constant internal pressure

Section shape after optimization: circular



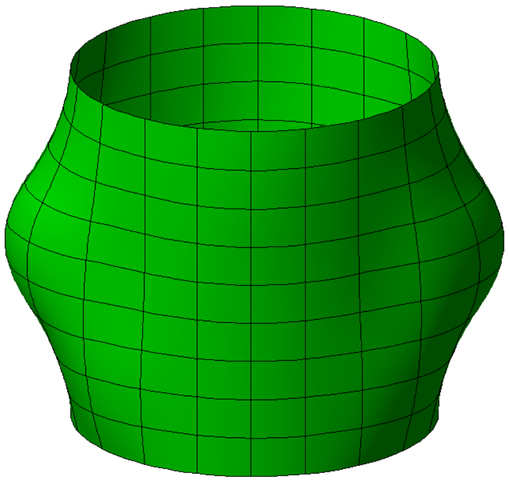
Circular section exactly represented by NURBS

Example: Circular Tube under point loads

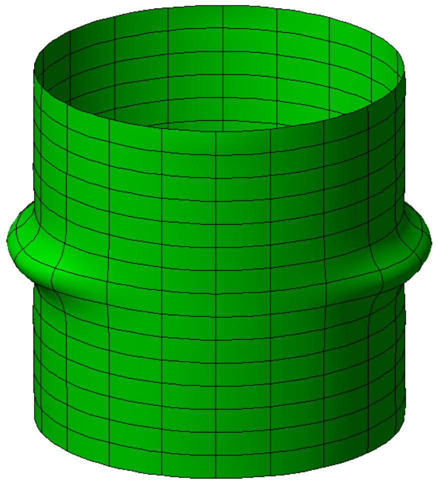


Example: Circular Tube under point loads

Different refinements for optimization model



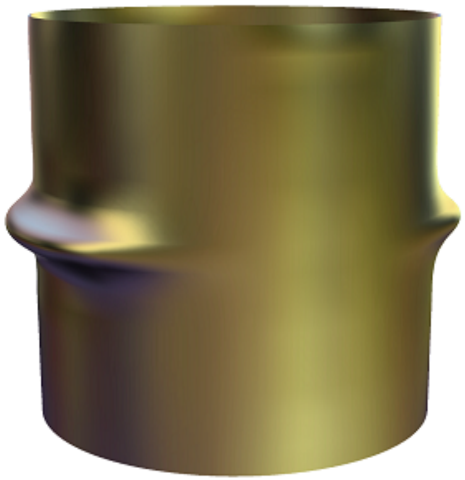
coarse optimization model



fine optimization model

Example: Circular Tube under point loads

Optimized design model in CAD program



Thank you for your attention