Isogeometric Analysis for Shell Structures

Josef Kiendl
Isogeometric Analysis for Shell Structures

Outline

- Introduction to shells
- Kirchhoff-Love shell theory
- Isogeometric shell analysis
- Integration into CAD
- Application to FSI simulations of wind turbine blades
- Isogeometric shape optimization
Introduction – What is a Shell?

A shell is a curved three-dimensional, thin-walled structure. It can be represented by a curved surface with a certain thickness.

Due to the curvature, shells can carry the load mostly by membrane forces, reducing bending moments. This implies a very efficient use of the material and allows to be very thin and light.

\[
\text{slenderness} = \frac{\text{radius of curvature}}{\text{thickness}} = \frac{R}{t}
\]
Shells in nature

Example:
The Egg

slenderness = \frac{\text{radius of curvature}}{\text{thickness}} = \frac{R}{t} \approx 60
Shells in nature

- seashell
- nutshell
- human skull
Shells in Civil Engineering and Architecture

Cupolas and vaults

Pantheon, Rom, 27 b.c.
D = 44m, R/t = 18

Taj Mahal, India

St. Paul's, London, 1506
R/t = 18
Shells in Civil Engineering and Architecture

modern cooling tower
R/t >> 500

concrete, R/t = 520
Kresge Hall, MIT

cement, R/t >> 500
Heinz Isler

concrete roof, Heinz Isler
Shells in automotive engineering

pictures:
DaimlerChrysler
BMW
Jaguar
Shells in naval and aerospace engineering

pictures:
Aida
Airbus
Saab
Space Shuttle
Shells in everyday‘s life

pictures: Braun, CocaCola, Hyve
Shell Theories

Shell models can be classified into:

• Kirchhoff–Love, classical shell theory

• Reissner–Mindlin, shear deformable theory

• Higher order formulations: thickness-deformable, multilayer, multidirector
Kirchhoff-Love

Kinematic assumptions:

- Thickness remains unchanged $\varepsilon_{zz} = 0$
- Cross sections remain straight
- Cross sections remain normal
Reissner-Mindlin

Kinematic assumptions:

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- Cross sections remain straight
- Cross sections remain normal

$\gamma_{xz} \rightarrow$ transverse shear strain
Kirchhoff-Love

Kinematic assumptions:

- Thickness remains unchanged $\varepsilon_{zz} = 0$
- Cross sections remain straight
- Cross sections remain normal

$\gamma_{xz} = 0$ -> transverse shear strain
Strains

Kirchhoff assumptions  -> no strains in thickness direction  $\varepsilon_{i3} = \varepsilon_{3i} = 0$

-> in-plane strains linear through thickness

Separating in-plane strains into constant and symmetrically linear part

- Constant part = membrane strain $\varepsilon_{\alpha\beta}$
- Symmetric linear part = bending strain
  -> $\kappa_{\alpha\beta}$ change in curvature

$$E_{\alpha\beta} = \varepsilon_{\alpha\beta} + \theta^3 \kappa_{\alpha\beta}$$

$$\varepsilon_{\alpha\beta} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} \end{bmatrix} \quad \kappa_{\alpha\beta} = \begin{bmatrix} \kappa_{11} & \kappa_{12} \\ \kappa_{21} & \kappa_{22} \end{bmatrix}$$
Geometric non-linearity

- **Geometrically linear:**
  - linear relation between strain and deformation
  - strain obtained as derivatives of displacements
    \[ \varepsilon_x = \frac{\partial u_x}{\partial x} \quad \kappa_x = -\frac{\partial^2 w}{\partial x^2} \]
  -> valid only for small deformations

- **Geometrically non-linear:**
  - non-linear relation between strain and deformation
  - various non-linear strain measures
  - study of geometry in deformed and undeformed configuration

-> valid for small and large deformations
Geometric non-linearity

Study of geometry in deformed/actual and undeformed/reference configuration

\[ \mathbf{u} = \mathbf{x} - \mathbf{X} \]

- \( \mathbf{u} \) - displacement vector
- \( \mathbf{x} \) - position vector actual configuration
- \( \mathbf{X} \) - position vector reference configuration
- \( \mathbf{g}_i \) - local base vectors actual configuration
- \( \mathbf{G}_i \) - local base vectors reference configuration
- \( g_{ij} = \mathbf{g}_i \cdot \mathbf{g}_j \) - metric tensor actual configuration
- \( G_{ij} = \mathbf{G}_i \cdot \mathbf{G}_j \) - metric tensor reference configuration
**Deformation gradient**

Deformation gradient tensor:

\[ F = \frac{dx}{dX} \]

\[ dx = F \cdot dX \]

\[ \text{dx} \text{ - infinitesimal vector actual configuration} \]
\[ dX \text{ - infinitesimal vector reference configuration} \]

\[ F = g_i \otimes G^i \]

-> The deformation gradient tensor contains all information relative to local strain.
-> However, contains rigid body motions -> no objective strain measure
Strain and stress tensors

Deformation gradient tensor: \( \mathbf{F} = g_i \otimes G^i \)

Euler-Almansi strain tensor: \( \mathbf{e} = \frac{1}{2}(\mathbf{I} - \mathbf{F}^{-T}\mathbf{F}^{-1}) \)

Green-Lagrange strain tensor: \( \mathbf{E} = \frac{1}{2}(\mathbf{F}^T\mathbf{F} - \mathbf{I}) \) \( \mathbf{I} \) - identity tensor

\[ E_{ij} = \frac{1}{2}(g_{ij} - G_{ij}) \] \( g_{ij}, G_{ij} \) - metric coefficients

Second Piola-Kirchhoff stress tensor: \( \mathbf{S} = \mathbf{C} : \mathbf{E} \) \( \mathbf{C} \) - material tensor

\[ \mathbf{S} = \det\mathbf{F} \cdot \mathbf{F}^{-1} \cdot \mathbf{\sigma} \cdot \mathbf{F}^{-T} \]

\( \mathbf{\sigma} \) - Cauchy stress tensor
Strain and stress tensors

Deformation gradient tensor: \( \mathbf{F} = \mathbf{g}_i \otimes \mathbf{G}^i \)

Euler-Almansi strain tensor: \( \mathbf{e} = \frac{1}{2}(\mathbf{I} - \mathbf{F}^{-T}\mathbf{F}^{-1}) \)

Green-Lagrange strain tensor: \( \mathbf{E} = \frac{1}{2}(\mathbf{F}^T\mathbf{F} - \mathbf{I}) \) \( \mathbf{I} \) - identity tensor

\[ E_{\alpha\beta} = \frac{1}{2}(g_{\alpha\beta} - G_{\alpha\beta}) \]
\( g_{\alpha\beta}, G_{\alpha\beta} \) - metric coefficients

Second Piola-Kirchhoff stress tensor: \( \mathbf{S} = \mathbf{C} : \mathbf{E} \) \( \mathbf{C} \) - material tensor

\[ \mathbf{S} = \det \mathbf{F} \cdot \mathbf{F}^{-1} \cdot \mathbf{\sigma} \cdot \mathbf{F}^{-T} \]
\( \mathbf{\sigma} \) - Cauchy stress tensor
From continuum to shell kinematics

Shell model = surface model

=> relate all quantities of the shell continuum to its middle surface

Shell continuum:

\[ x \]

\[ g_{\alpha} = \frac{\partial x}{\partial \theta^\alpha} = x_{,\alpha} \quad \alpha = \{1, 2\} \]

Shell middle surface:

\[ a_{\alpha} = g_{\alpha} (\theta^3 = 0) \]

\[ a_3 = \frac{a_1 \times a_2}{|a_1 \times a_2|} \]

\[ x = \theta^\alpha a_{\alpha} + \theta^3 a_3 \]

\[ g_{\alpha} = a_{\alpha} + \theta^3 a_{3,\alpha} \]

Metric tensor:

\[ g_{\alpha\beta} = a_{\alpha\beta} - 2 \theta^3 b_{\alpha\beta} + (\theta^3)^2 a_{3,\alpha} \cdot a_{3,\beta} \]

Strain tensor:

\[ E_{\alpha\beta} = \frac{1}{2} (a_{\alpha\beta} - A_{\alpha\beta}) + \theta^3 (B_{\alpha\beta} - b_{\alpha\beta}) \]

metric tensor of the midsurface "1\textsuperscript{st} fundamental form"

curvature tensor of the midsurface "2\textsuperscript{nd} fundamental form"

neglect quadratic term for linear strain distribution
Strains

Separating in-plane strains into constant and symmetrically linear part

\[ E_{\alpha\beta} = \varepsilon_{\alpha\beta} + \theta^3 \kappa_{\alpha\beta} \]

- Constant part = membrane strain \( \varepsilon_{\alpha\beta} \)
- Symmetric linear part = bending strain
  \( \rightarrow \) \( \kappa_{\alpha\beta} \) change in curvature

\[ E_{\alpha\beta} = \frac{1}{2}(a_{\alpha\beta} - A_{\alpha\beta}) + \theta^3(B_{\alpha\beta} - b_{\alpha\beta}) \]

\[ \varepsilon_{\alpha\beta} = \frac{1}{2}(a_{\alpha\beta} - A_{\alpha\beta}) \rightarrow \text{Membrane strain from metric tensors} \]

\[ \kappa_{\alpha\beta} = B_{\alpha\beta} - b_{\alpha\beta} \rightarrow \text{Bending strain from curvature tensors} \]
Variational Formulation

Principle of virtual work

Variation $\delta u_r$

$$\delta W = \frac{\partial W}{\partial u_r} \delta u_r = 0$$

Linearization

$$\frac{\partial W}{\partial u_r} + \frac{\partial^2 W}{\partial u_r \partial u_s} \Delta u_s = 0$$

$$\mathbf{R} - \mathbf{K} \Delta \mathbf{u} = 0$$

$\mathbf{R}$ - residual force vector
$\mathbf{K}$ - stiffness matrix
$\Delta \mathbf{u}$ - displacement vector
Variational Formulation

Internal virtual work
\[ \delta W_{int} = - \int_{\Omega} (S : \delta E) \, d\Omega \]

- \( S \) - Piola-Kirchhoff 2 stress tensor
- \( E \) - Green-Lagrange strain tensor

=> Separating strain into membrane and bending action
- \( \varepsilon \) - membrane strain
- \( n \) - normal forces
- \( \kappa \) - change of curvature
- \( m \) - bending moments

\[ \begin{align*}
\delta W_{int} &= - \int_{A} (n : \delta \varepsilon + m : \delta \kappa) \, dA \\
E_{\alpha\beta}(\theta^3) &= \varepsilon_{\alpha\beta} + \theta^3 \kappa_{\alpha\beta}
\end{align*} \]

=> Problem for FE: curvature -> 2nd derivatives -> \( C^1 \) continuity between elements
Isogeometric Analysis for KL Shell formulation

NURBS-based IGA: -> Exact geometry in analysis model
⇒ C¹ and higher continuity between elements provided by NURBS
⇒ exact evaluation of geometric properties including curvatures
⇒ straightforward implementation of kinematic formulation; no further assumptions
⇒ no rotational degrees of freedom
⇒ no shear locking!

\[ S(u, v) = \frac{\sum_{j=1}^{m} \sum_{i=1}^{n} N_{i,p}(u) \cdot M_{j,q}(v) \cdot w_{i,j} \cdot P_{i,j}}{\sum_{j=1}^{m} \sum_{i=1}^{n} N_{i,p}(u) \cdot M_{j,q}(v) \cdot w_{i,j}} \]
Rotational boundary conditions

Open knot vector: $U = \{0, 0, 0, 0, 0.25, 0.5, 0.75, 1, 1, 1, 1\}$

$\text{B-Spline basis functions}$

$\text{First derivatives of B-Spline basis functions}$

$\rightarrow$ first and last control point points are interpolated

$\rightarrow$ second and next to last control point define the curve’s tangents at the ends
Rotational boundary conditions

Needed for: clamped support, symmetry conditions

-> Tangent at the boundary is determined by 1\textsuperscript{st} and 2\textsuperscript{nd} row of control points

Example: cantilever plate
Benchmarking

Shell obstacle course

- Pinched hemisphere (1/4)
- Pinched cylinder (1/8)
- Scordelis-Lo roof (1/4)
Benchmarking

Shell obstacle course

pinched hemisphere (1/4)

pinched cylinder (1/8)

Scordelis-Lo roof (1/4)
Shell obstacle course – convergence charts

Hemispherical shell
Pinched cylinder
Scordelis-Lo roof
Benchmarking

Shell obstacle course

setup

deformations

pinched hemisphere  pinched cylinder  Scordelis-Lo roof
Large deformations and rotations

Benchmark: Straight plate bent to a circle

Moment \( M = \frac{2\pi EI}{L} \) modeled by a pair of forces perpendicular to the geometry
Large Deformations / Large Rotations

Example 2: Cantilever beam subject to constant twisting moment.

Moment \( M_t = \frac{\theta G I}{L} \) modeled by a pair of forces perpendicular to the geometry
Multiple Patches

NURBS patches are $C^0$ on the boundary

Smooth Patches:

Non-smooth Patches:
**Multiple Patches**

NURBS patches are $C^0$ on the boundary

**Smooth Patches:**

\[
P_2^2 = P_1^1 = P_{n-1}^1
\]

**Non-smooth Patches:**

\[
\alpha = \cos^{-1}\left(\frac{(P_n^1 - P_{n-1}^1) \cdot (P_2^2 - P_1^1)}{|P_n^1 - P_{n-1}^1| \cdot |P_2^2 - P_1^1|}\right)
\]

**Constraint equation:**

\[
P_2^2 = (1 + c) P_n^1 - c P_{n-1}^1
\]
Smooth, $C^1$-continuous, Patches

Coupling of control points across patch boundaries
-> continuity constraint is fulfilled exactly
Patches forming a kink

-> Maintain angle between patches. Not possible by direct coupling of control points!

$C^0$ coupling: no transfer of bending moments  =>  Connection acts as a hinge

L-profile cantilever
Bending Strips

Couple control points defining the angle between patches by a stiff bending strip

- High bending stiffness transversal to the patch interface
- Zero membrane stiffness, zero bending stiffness along the interface
- Zero mass

=> no additional stiffness to the system, only constraining the angle between the patches
Bending Strips

L-profile cantilever

solution without bending strip

solution with bending strip
Bending Strips for smooth patches

Scordelis-Lo roof modeled by two patches

$C^0$ connection => kink

Correct solution with bending strip
Scordelis-Lo roof – stress resultant plots

Stress resultant

Isogeometric K-L shell

Bending strip solution

n1

m1

q1

-3510

-93

-280

+127

+2079

+280
Bending Strips

-> Applicable to both smooth patches and patches with kink

Scordelis-Lo roof – deformation plots

single patch
two patches => kink
kink repaired by bending strip

=> simple and efficient method to treat arbitrary multipatch structures
Shell with stiffener (Rank et al.)

Bending Strips for coupling of Shells and Solids

Shell modeled by shell elements
Stiffener modeled by solid elements
coupled by a bending strip
Outline

- Introduction
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- Isogeometric shape optimization
CAD descriptions are surface-based

- No volumetric NURBS description in CAD -> creating volumetric model from surface model necessary
- For thin-walled structures: shell analysis on the surface model
- Rotation-free shell => surface model from CAD program can be used for analysis without modification
Rhinoceros

NURBS-based CAD software
for industrial, mechanical, marine, architectural design, etc.
allows for self-written plug-ins -> plug-in for isogeometric analysis

sail cutter
www.viribusunitis.ca

microcupyacht
www.flexicad.com

Opel Astra
www.rhino3d.com
Rhinoceros

NURBS-based CAD software
for industrial, mechanical, marine, architectural design, etc.

Plug-in for isogeometric analysis
Robert Schmidt
Michael Breitenberger

IGA_movie1.mp4     IGA_movie2.mp4
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Application: FSI simulation of a Wind Turbine Blade

Cooperation project with Y. Bazilevs, M.-C. Hsu, University of California, San Diego

5MW offshore wind turbines:
- Blade radius: 63 m
- Blade material: fibreglass
  \[ E_1 = 43.3 \text{GP} \quad E_2 = 12.7 \text{GP} \quad G_{12} = 4.5 \text{GP} \quad \nu_{12} = 0.29 \]
- Speed of rotation: 8-13 RPM
- Wind speed: 8~12 m/s
- Reynolds number: Several hundred million
Fluid and Mesh Motion

- Fluid:
  - incompressible Newtonian fluid in the ALE description
  - turbulence modeling: residual based variational multiscale method (RBVMS)
  - NURBS as basis functions

- Mesh Motion:
  - divided into rotation (exactly) and blade deflection (linear elasticity)

- Kinematic (strong) and traction (weak) compatibility at the fluid-solid boundary
Problem Description

- Full scale simulation
- NURBS-based Isogeometric Analysis
- 1,111,968 quadratic NURBS elements
- 60 processors
- Symmetry condition
Blade Design: Airfoils

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<th>Chord (m)</th>
<th>AeroCent (-)</th>
<th>OrigAC (-)</th>
<th>Airfoil</th>
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Blade-pitch axis

OrigAC

RN0des

Chord

X/Chord

(0.25-AeroCent)
Blade Design: NURBS
3D FSI Simulation
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Shape Optimization

• Optimization Problem:
  - objective function: \( f(s) \rightarrow \min \) e.g.: \( 0.5 \cdot \int \sigma \varepsilon \, dV \rightarrow \min \)
  - equality constraints: \( h(s) = 0 \) e.g. mass: \( m = m_{\text{def}} \)
  - inequality constraints: \( g(s) \leq 0 \) e.g. stress: \( \sigma \leq \sigma_{\text{max}} \)
  - design variables: \( s_i, \quad i = 1, \ldots, n \) shape variables e.g. \( l, r, x_i, y_i, z_i \)

• Shape Optimization with FE \( \rightarrow \) two different geometry descriptions

• Design Parametrization:
  - CAD-based: CAD parameters
  - FE-based: FE nodes

Isogeometric Shape Optimization
**Shape Optimization**

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Shape Optimization

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  \[ \text{e.g.: } 0.5 \cdot \int \sigma \varepsilon \, dV \rightarrow \min. \]
  \[ \text{e.g. mass: } m = m_{\text{def}} \]
  \[ \text{e.g. stress: } \sigma \leq \sigma_{\text{max}} \]
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Shape Optimization

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- Shape Optimization with FE -> two different geometry descriptions

- Design Parametrization:
  - CAD-based: CAD parameters
  - FE-based: FE nodes
  - **Isogeometric: Control Points**
Isogeometric Shape Optimization

Design variables: Control points

- CAD model
- Isogeometric analysis model
- Isogeometric optimization model
Isogeometric Shape Optimization

Example: Tube under constant internal pressure

Find optimal shape of the section in order to maximize the stiffness
Example: Tube under constant internal pressure

12 design variables: x, y, w of the four edge control points
Animation: Optimization steps
Example: Tube under constant internal pressure

Section shape after optimization: circular

Circular section exactly represented by NURBS
Example: Circular Tube under point loads
Example: Circular Tube under point loads

Different refinements for optimization model

coarse optimization model

fine optimization model
Example: Circular Tube under point loads

Optimized design model in CAD program
Thank you for your attention