

EXERCISE 1

- Write a Matlab function `[x,mu]=power(A,maxit,tol)` that implements the power method to approximate the dominant eigenvalue of a matrix (use a random vector as initial guess). Test it on the matrix

$$A = Q*\text{diag}(1:10)*\text{inv}(Q), \text{ with } Q = \text{orth}(\text{randn}(10,10))$$

Note that the spectrum of A is $\{1, \dots, 10\}$. Modify the function to be able to plot the relative error $|\lambda_1 - \mu_k| / |\lambda_1|$ and also $(|\lambda_2/\lambda_1|)^k$, $k = 1, 2, \dots$. What do you observe?

- Same as above, but test on the nonsymmetric matrix obtained using $Q = \text{randn}(10,10)$. Compare the results with the previous case.
- Write another function, `inverse_power`, that implements the inverse power method (use “backslash” to solve the linear system), and test it on the matrix used in the previous point to approximate $\lambda_9 = 2$ using

$$\mu = 1.55, 1.65, 1.75, 1.85, 1.95.$$

Plot the number of iterations required to converge vs. the value $|\lambda_9 - \mu| / |\lambda_{10} - \mu|$ and comment.