Numerical Methods in Engineering Sciences
20/7/2022

Written Exam

First name: _____
Last name: _____

Student ID: _____

□ I want to take the BASIC EXAM
□ I want to take the ADVANCED EXAM

Exam rules:

- Basic exam: the maximum grade is 24/30.

- Advanced exam: the maximum grade is 30/30 cum laude.

Total time is 1 hour. Students who get a positive grade in the written part (i.e., at least 18/30) *might* choose to take an oral exam. For students who choose the basic written exam, the maximum grade obtainable can never exceed 24/30.

BASIC EXAM

1. Starting from $x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, compute 2 iterations of the Jacobi method to compute an approximate solution of the system $Ax = b$, where

$$A = \begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 1 \\ 1 & -1 & 4 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

2.
Introduce the power method for the computation of the dominant eigenvalue and eigenvector, with pseudocode. State conditions that guarantee its convergence

ADVANCED EXAM

3. Write the pseudo-code of the composite midpoint quadrature rule, then use the composite midpoint quadrature rule to compute an approximation of

$$\int_{-1}^{2} (t^2 + 2t)\, dt$$

by splitting the integration interval $[-1, 2]$ into three subintervals. Report the intermediate computations.

4. State and prove the theorem on the existence and uniqueness of the Lagrange interpolant of a given function.