

- I want to take the BASIC EXAM
 I want to take the ADVANCED EXAM

Exam rules:

- Basic exam: the maximum grade is 24/30.
- Advanced exam: the maximum grade is 30/30 cum laude.

Total time is 1 hour. Students who get a positive grade in the written part (i.e., at least 18/30) *might* choose to take an oral exam. For students who choose the basic written exam, the maximum grade obtainable can never exceed 24/30.

BASIC EXAM

1. Write the LU factorization, without pivoting, of:

$$\begin{bmatrix} 2 & 7 & 5 \\ 14 & 50 & 36 \\ 2 & 8 & 8 \end{bmatrix}$$

showing the intermediate computations.

2. Write the pseudo-code of the composite Simpson quadrature rule, then use it to compute an approximation of

$$\int_{-1}^1 (1 + 2t + t^2) dt$$

by splitting the integration interval $[-1, 1]$ into two subintervals. Report the intermediate computations.

ADVANCED EXAM

3. Given the matrix

$$\begin{bmatrix} \alpha & 0 & 1 \\ 0 & \alpha & 1 \\ 1 & 1 & \alpha \end{bmatrix}, \quad \alpha \in \mathbb{R}^+,$$

determine for which values of α the Gauss-Seidel method applied to the system $Ax = b$ converges for every $b \in \mathbb{R}^3$ and for every initial guess $x^{(0)} \in \mathbb{R}^3$.

4. Give the statement of the convergence theorem of the Newton method for nonlinear equations. Prove that the order of convergence of the methods is 2.