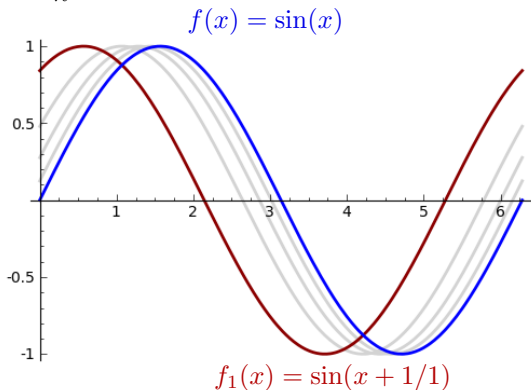


Capitolo 2: Serie di funzioni

March 6, 2012

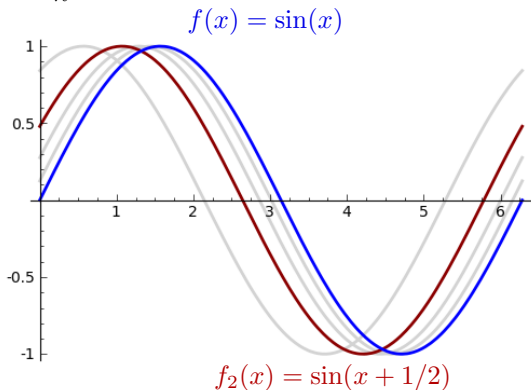
2.1 Successioni di funzioni

$$f_n(x) = \sin\left(x + \frac{1}{n}\right)$$



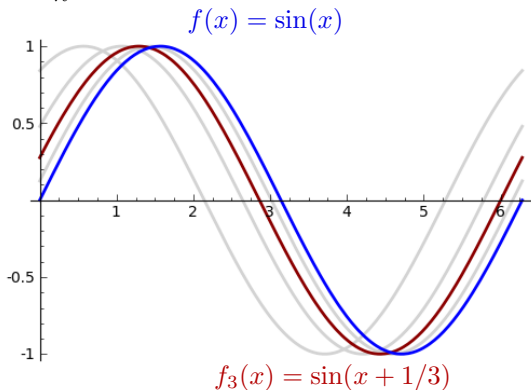
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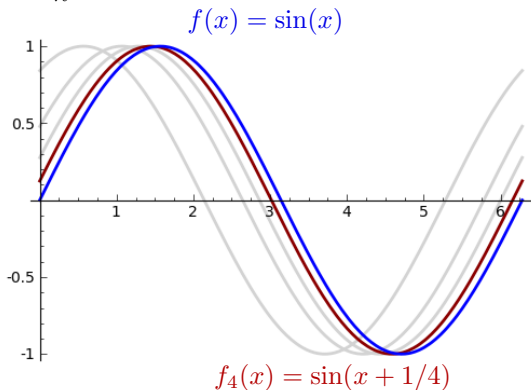
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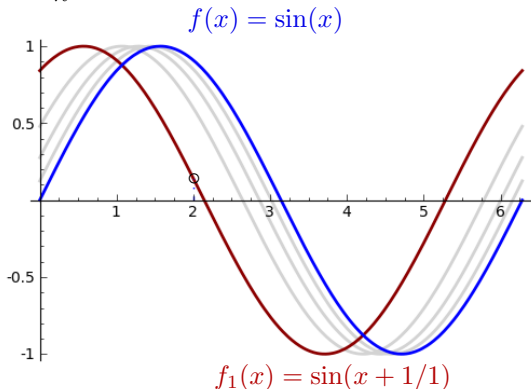
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2.1 Successioni di funzioni

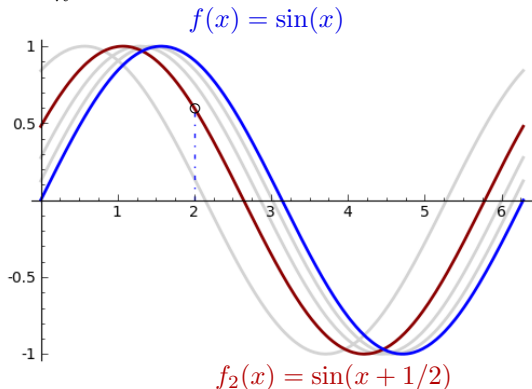
$$f_n(x) = \sin\left(x + \frac{1}{n}\right)$$



convergenza *puntuale*, e.g. $f_n(2) \rightarrow f(2)$

2.1 Successioni di funzioni

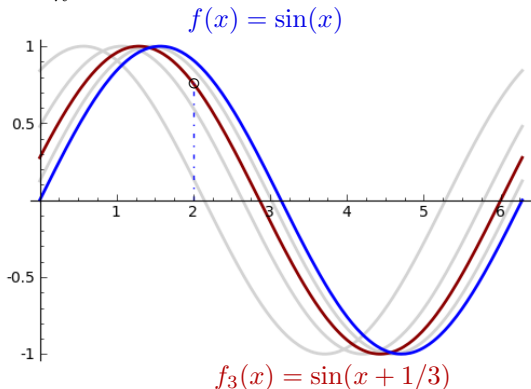
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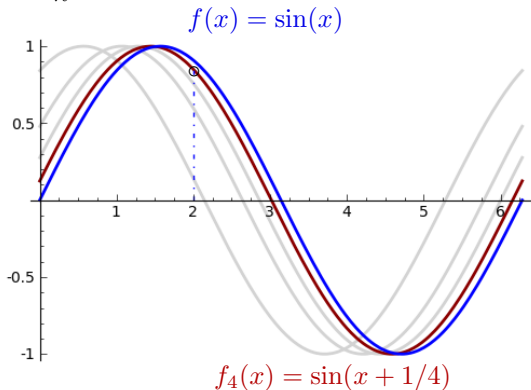
$$f_n(x) = \sin\left(x + \frac{1}{n}\right)$$



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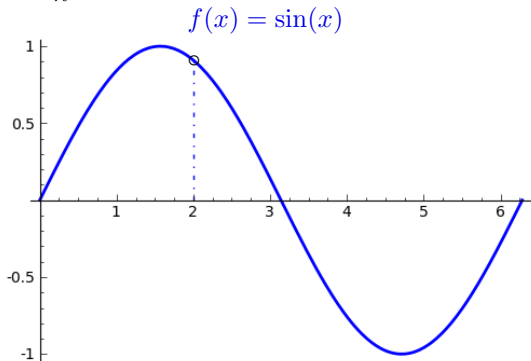
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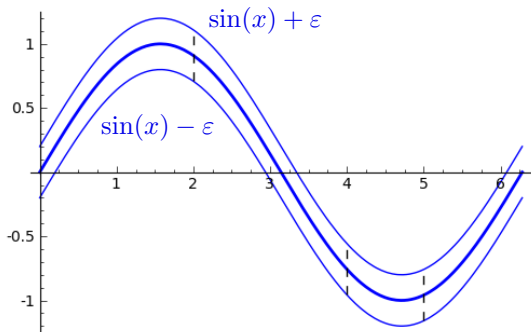
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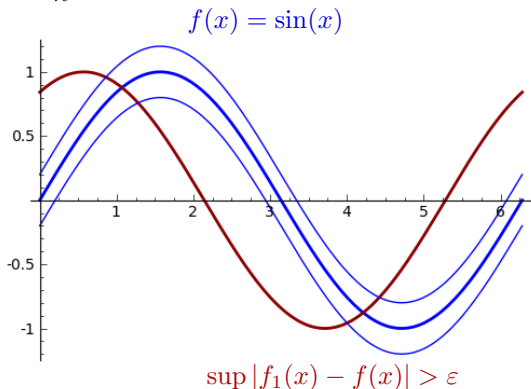
$$f_n(x) = \sin\left(x + \frac{1}{n}\right)$$



convergenza *uniforme*: $\forall \epsilon > 0 \exists \bar{n} \in \mathbb{N} : \forall n > \bar{n} \sup_x |f_n(x) - f(x)| \leq \epsilon$

2.1 Successioni di funzioni

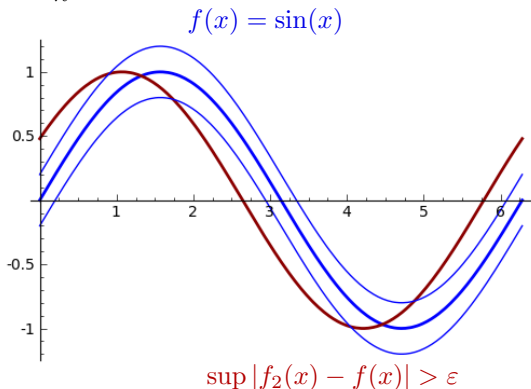
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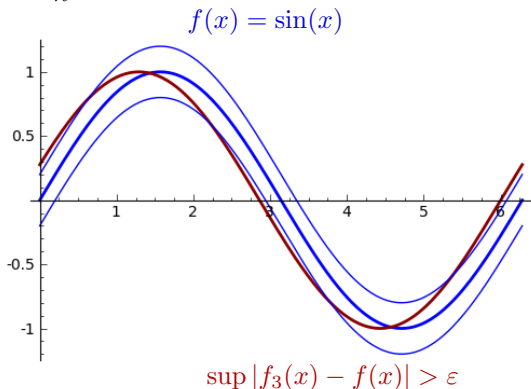
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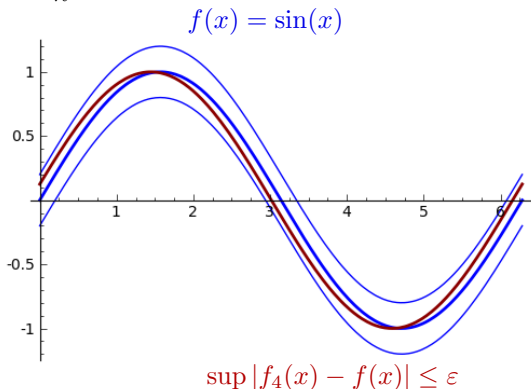
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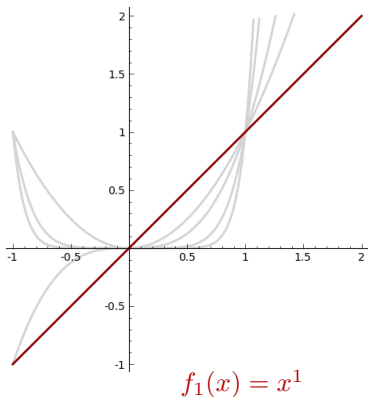
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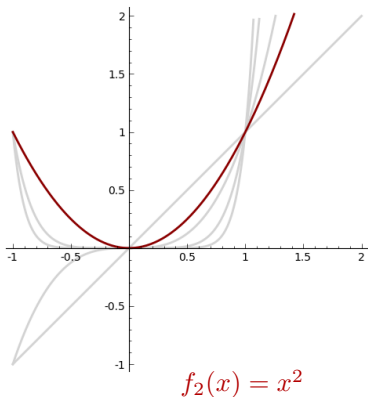
$$f_n(x) = x^n$$



f_n converge a ?

2.1 Successioni di funzioni

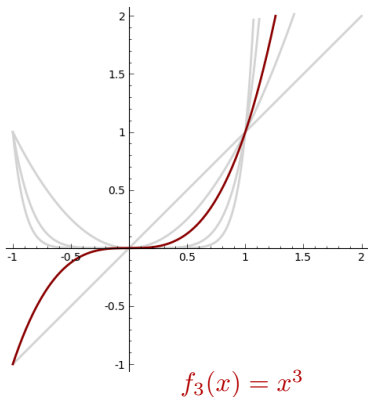
$$f_n(x) = x^n$$



f_n converge a ?

2.1 Successioni di funzioni

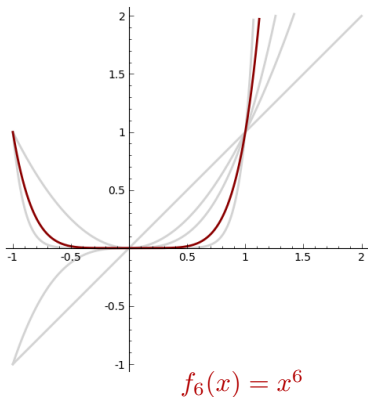
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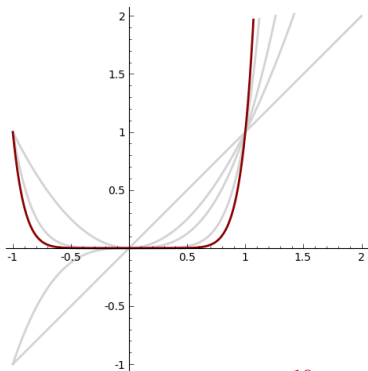
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2.1 Successioni di funzioni

$$f_n(x) = x^n$$



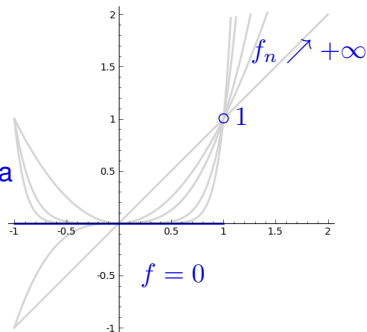
$$f_{10}(x) = x^{10}$$

f_n converge a ?

2.1 Successioni di funzioni

$$f_n(x) = x^n$$

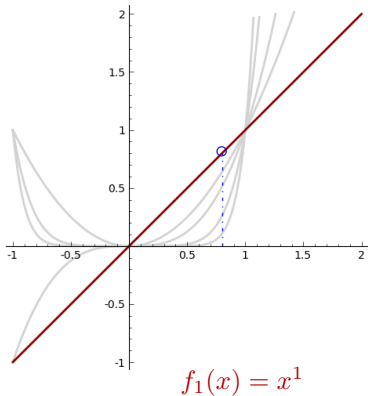
f_n indeterminata
 $x \leq -1$



f_n converge puntualmente a $f = \begin{cases} 0 & -1 < x < 1 \\ 1 & x = 1 \\ +\infty & x > 1 \end{cases}$

2.1 Successioni di funzioni

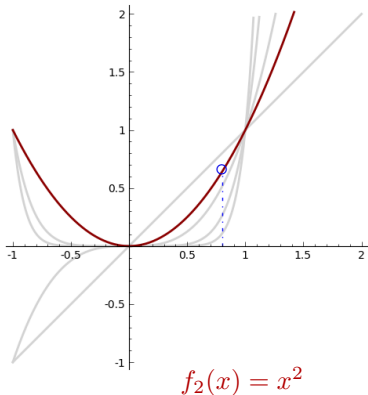
$$f_n(x) = x^n$$



convergenza *puntuale*, e.g. $f_n(0.8) \rightarrow 0$

2.1 Successioni di funzioni

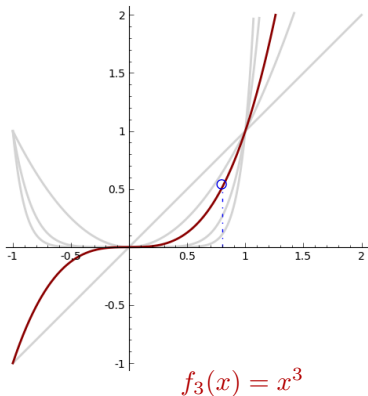
$$f_n(x) = x^n$$



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2.1 Successioni di funzioni

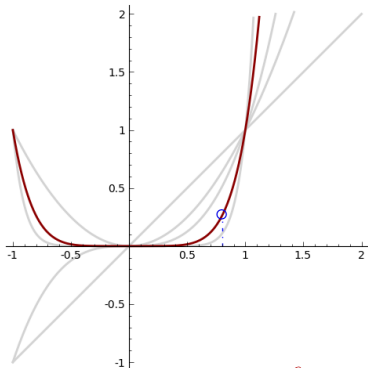
$$f_n(x) = x^n$$



convergenza *puntuale*, e.g. $f_n(0.8) \rightarrow 0$

2.1 Successioni di funzioni

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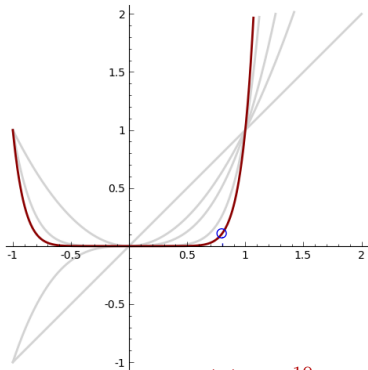


$$f_6(x) = x^6$$

convergenza *puntuale*, e.g. $f_n(0.8) \rightarrow 0$

2.1 Successioni di funzioni

$$f_n(x) = x^n$$

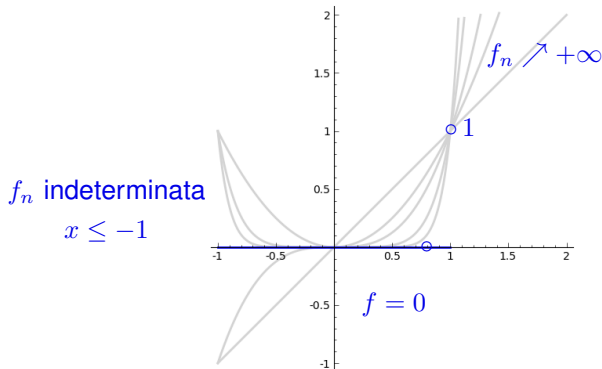


$$f_{10}(x) = x^{10}$$

convergenza *puntuale*, e.g. $f_n(0.8) \rightarrow 0$

2.1 Successioni di funzioni

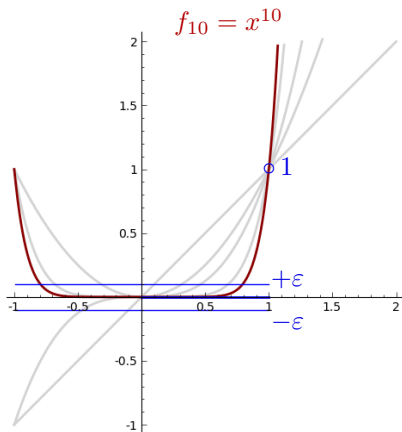
$$f_n(x) = x^n$$



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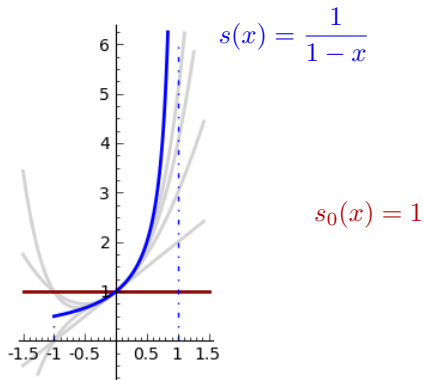
$$f_n(x) = x^n$$



f_n NON converge uniformemente su $[-1, 1]$, (ma su $[-\alpha, \alpha]$, $0 < \alpha < 1$)

2.4 Serie di potenze

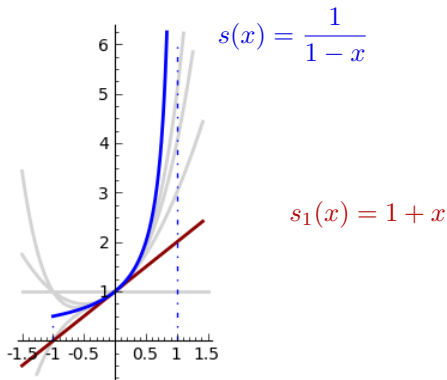
$$f_n(x) = x^n \quad s_n(x) = \sum_{k=0}^n f_k(x) = 1 + x + x^2 + \dots + x^n$$



Convergenza puntuale: $\sum_{k=0}^{+\infty} x^k = \lim_{n \rightarrow +\infty} s_n(x) = s(x) \quad \forall x \in (-1, 1)$

2.4 Serie di potenze

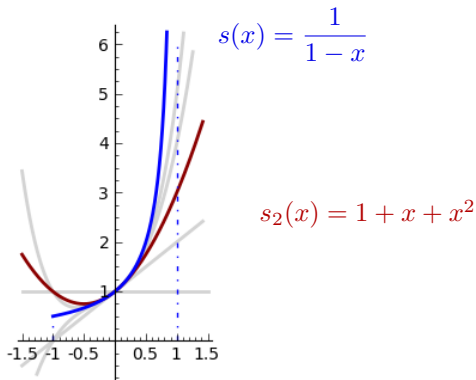
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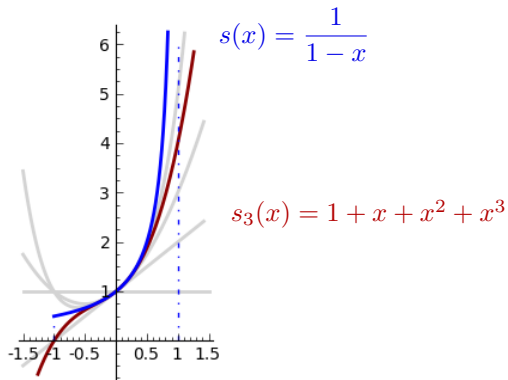
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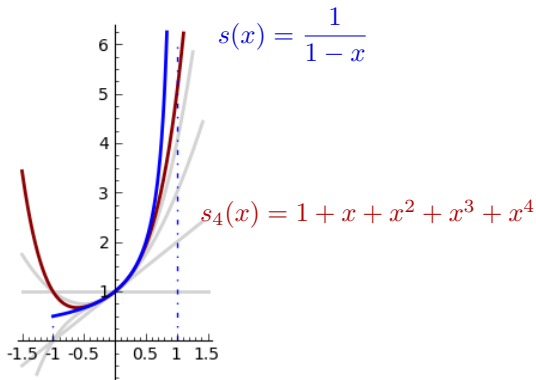
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Convergenza puntuale: $\sum_{k=0}^{+\infty} x^k = \lim_{n \rightarrow +\infty} s_n(x) = s(x) \quad \forall x \in (-1, 1)$

Informazioni tecniche:

- Le figure sono state realizzate con *Sage*

<http://www.sagemath.org/>

- Le slides sono state realizzate con \LaTeX

<http://www.latex-project.org/>

(Entrambi sono free software)