$\boxed{A}$

1. Per $n=1,2,3,\dots$ $\cos(n\pi) = -1, 1, -1, \dots$

- $\lim_{n\to+\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to+\infty} \frac{n+1}{n} = 1$ $\quad R=1 \quad x_0 = 3$
- In $x=2$ $\quad \sum \frac{\cos(n\pi)}{n}(x-3)^n = \sum \frac{(-1)^n}{n}\cdot(-1)^n = +\infty$

In $x=4$ $\quad \sum \frac{\cos(n\pi)}{n}(x-3)^n = \sum \frac{(-1)^n}{n}\cdot 1^n = -\ln(2) = (b$

$\left(\text{Dato che } \ln(1+x) = \sum \frac{(-1)^{n+1}}{n} x^n\right)$. $\Rightarrow (a) =]2,4]$.

2. (a) $\partial_x f = 8\cdot 5\cdot(4x-3)^7\cdot 4$

$\partial_y f = -\sin(y)$

(b) I p.ti stazionari sono $\left(\frac{3}{4}, k\pi\right)$, $k\in$

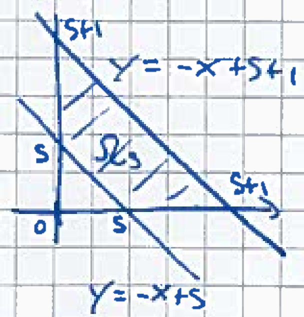
In $(-1,4)\times(-1,4)$ ci sono solo $A=\left(\frac{3}{4},0\right)$ e $B=\left(\frac{3}{4},\pi\right)$

(c) Esaminando i grafici di $x \mapsto 5(4x-3)^8$ e $y \mapsto \cos(y)$ si vede che A è p.to di sella (min. per x e max. per y) mentre B è p.to di min. assoluto.

3. $\gamma'(t) = \left(-4\sin t, 4\cos t, 3t/2\right)$, $\quad |\gamma'|^2 = 16(\sin^2 t+\cos^2 t) + \frac{9t^2}{4}$

$$\int_\gamma f\, ds = \int_0^1 f(\gamma(t))\,|\gamma'(t)|\,dt$$
$$= \int_0^1 \left(16+9t^2/4\right)^{\frac{1}{2}}\cdot\left(16+9t^2/4\right)^{\frac{1}{2}} dt$$
$$= \int_0^1 16 + 9t^2/4\, dt = \left[16t + \frac{3}{4}t^3\right]_0^1 = 16+\frac{3}{4} = \frac{67}{4}$$

4.



(a) $|\Omega_s|$ = area triangolo grande - area tr. piccolo

$= \frac{(s+1)(s+1)}{2} - \frac{s\cdot s}{2} = s+\frac{1}{2}$

(b) $|V| = \int_0^2 \left(\iint_{\Omega_z} 1\, dxdy\right) dz = \int_0^2 \left(z+\frac{1}{2}\right) dz = 3$

5. $F(x, g(x)) = 0$ in un intorno di $(0,0)$ significa

(1) $g(0) = 0$ (2) $e^{3xg(x)} + 5x^2 + 2g(x) - 1 = 0$

Derivando (2) si ottiene:

$$(3g + 3xg')e^{3xg} + 10x + 2g' = 0 \overset{(1)}{\Longrightarrow} g'(0) = 0 \quad (3)$$

Derivando ancora:

$$\left(3g' + 3g' + 3xg'' + (3g + 3xg')^2\right)e^{3xg} + 10 + 2g'' = 0$$

$$\Rightarrow g'' = \frac{-10}{2} = -5$$

$$P_2(x) = g(0) + g'(0)x + \frac{g''(0)}{2}x^2 = 0 + 0 - \frac{5}{2}x^2.$$

6. $\begin{cases} \nabla f = \lambda \nabla g \\ g = 0 \end{cases} \Rightarrow \begin{cases} 0 = 2\lambda x \\ -2y = 2\lambda y \\ 6 = 2\lambda z \\ x^2 + y^2 + z^2 - 36 = 0 \end{cases}$

La ~~prima~~ seconda equazione ha 2 soluzioni: I $\lambda = -1$ oppure $y = 0$ II

Da I ricavo: $\begin{cases} x = 0 \quad e \quad z = -3 \\ 0^2 + y^2 + 9 = 36 \end{cases} \Rightarrow \begin{array}{l} P = (0, 3\sqrt{3}, -3) \\ Q = (0, -3\sqrt{3}, -3) \end{array}$

Da II ricavo: $\begin{cases} x = 0 \\ 0^2 + 0^2 + z^2 = 36 \end{cases} \Rightarrow \begin{array}{l} A = (0, 0, 6) \\ B = (0, 0, -6) \end{array}$

Valutando f nei quattro punti, si trova: $\underline{f(A) = 36 \quad \max}$

$f(B) = -36$ $\underline{f(P) = f(Q) = -18 - 27 = -45 \quad \min}$.

7. $\partial_x f = \frac{\sinh(xy^2)}{\cosh(xy^2)} y^2 + 2$, $\partial_y f = 2xy\frac{\sinh(xy^2)}{\cosh(xy^2)} + 3$

$\Rightarrow$ il piano tg in $(0,0,1)$ è $z = 1 + 2x + 3y$.

$\boxed{D}$ non appartiene al piano: $1 + 2\cdot(2) + 3(-1) = 2 \neq 1$.

$\boxed{B}$

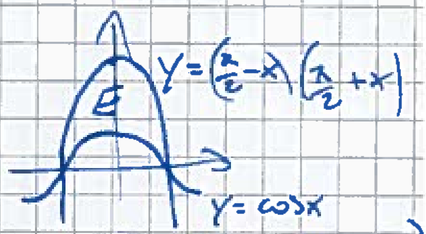
9. Per il teorema di Lagrange, esiste un punto c sul segmento che unisce $a=(0,0)$ e $b=(4,4)$, tale che

$$f(b)-f(a)=\nabla f(c)\cdot(b-a)\text{, cioè}$$

$$f(b)-5 = 4\,\partial_x f(c)+4\,\partial_y f(c) = 4\overbrace{\left(\partial_x f(c)+\partial_y f(c)\right)}^{1}$$

$$\Rightarrow f(b)=5+4=9.$$

11. (a) $\mathring{E}=E$



(b) $\overline{E}=\left\{y\ge\cos x,\ y\le\frac{\pi^2}{4}-x^2\right\}$

(c) $\partial E=\left\{x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right],\ y=\cos x\right\}\cup\left\{x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right],\ y=\frac{\pi^2}{4}-x^2\right\}$

12. E è aperto e limitato

13. $\operatorname{rot}F=(-\cos z,\,-\sin z,\,0)$

$$\Rightarrow F\cdot\operatorname{rot}F=-1 \quad\Rightarrow \iint_\Sigma F\cdot\operatorname{rot}F\,dS=-2\pi\neq 0.\ \boxed{A}$$

Inoltre: $\boxed{E}=\boxed{B}$ (Teo. di Stokes)

$$\boxed{B}=\int_0^{2\pi}(1,0,0)\cdot(-\sin t,\cos t,0)\,dt=0$$

$$\boxed{D}=\iiint_V \operatorname{div}F\,dx\,dy\,dz \quad \text{(Teo. di Gauss)}$$

e $\operatorname{div}F=0$.

$$\boxed{C}=\iiint_V \operatorname{div}(\operatorname{rot}F)\,dx\,dy\,dz \quad \text{(Teo di Gauss)}$$

e $\operatorname{div}(\operatorname{rot}F)=0$.