PARTE A

① Riscrivo la serie come:

$$\sum_{n=1}^{+\infty} \underbrace{\frac{1}{2} \cdot (-1)^{n+1} \cdot \frac{1}{n2^n}}_{a_n} \cdot \underbrace{(x+2)^n}_{x-x_0}$$

a) raggio $\lim_{n \to +\infty} \sqrt[n]{|a_n|} = \lim_{n \to +\infty} \frac{1}{\sqrt[n]{2n\,2^n}} = \frac{1}{2} \Rightarrow \boxed{R=2}$

b) $f^{(k)}(x_0) = a_k \cdot k! \Rightarrow f^{(2018)}(-2) = a_{2018} \cdot (2018!)$

$$= \frac{1}{2} \cdot (-1)^{2019} \cdot \frac{1}{2018 \cdot 2^{2018}} = \frac{-1}{2018 \cdot 2^{2019}} \cdot (2018!)$$

$$= -\frac{2017!}{2^{2019}}$$

c) Riscrivo la serie come:

$$\frac{1}{2} \sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n} \cdot \left(\frac{x+2}{2}\right)^n = \frac{1}{2} \ln\left(1 + \frac{x+2}{2}\right) = \frac{1}{2} \ln\left(\frac{x+4}{2}\right)$$

(2) $\partial_x f = y\cos(xy) - y + 4x$

$\partial_y f = x\cos(xy) - x + 2y$

$\Rightarrow \nabla f(0,0) = (0,0)$

$$Hf(0,0) = \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$$

$\Rightarrow$ $(0,0)$ è un p.to stazionario, di minimo locale.

(3) $\gamma'(t) = \left(e^{2t} - 2 \, , \, 2\sqrt{2}\, e^{t} \right)$

$|\gamma'(t)| = \sqrt{(e^{2t}-2)^2 + 8e^{2t}} = \sqrt{e^{4t} + 4e^{2t} + 4} = e^{2t} + 2$

$L = \int_0^1 |\gamma'(t)|\,dt = \int_0^1 e^{2t} + 2 = \left[\frac{e^{2t}}{2} + 2t \right]_{t=0}^{t=1} = \frac{e^2}{2} + \frac{3}{2}$

(4) $T = \{ x \in [0,1],\ 0 \leq y \leq 1-x \}$

$$\iint_T y e^{x+y}\, dx\,dy = \int_0^1 e^x \int_0^{1-x} y e^y\, dy\, dx$$

$$= \int_0^1 e^x \left[-\int_0^{1-x} e^y dy + [y e^y]_0^{1-x} \right] dx$$

$$= \int_0^1 e^x \left[1 - e^{1-x} + (1-x) e^{1-x} \right] dx$$

$$= \int_0^1 e^x - x \cdot e^1 \, dx = e^x - \frac{e}{2} x^2 \Big|_{x=0}^{x=1} = \frac{e}{2} - 1 .$$

⑤ Piano tg: $z = f(2,0) + \partial_x f(2,0)(x-2) + \partial_y f(2,0)(y-0)$

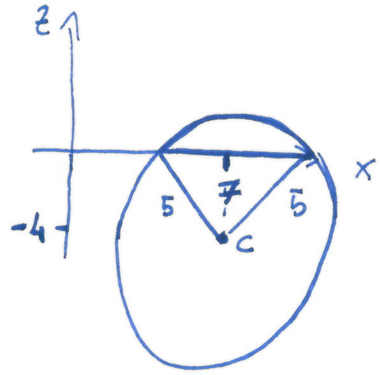
$f(2,0) = 0$

$\partial_x f(2,0) = \frac{d}{dx} f(x,0)\Big|_{x=2} = 0$

$\partial_y f(2,0) = \frac{d}{dy} f(2,y)\Big|_{y=0} = \frac{d}{dy} 4e^y \sin(2y)\Big|_{y=0} = 8$

Piano tg: $z = 8y$

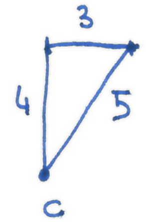
⑥ sezione:

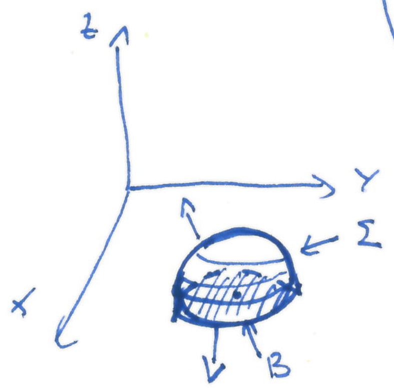


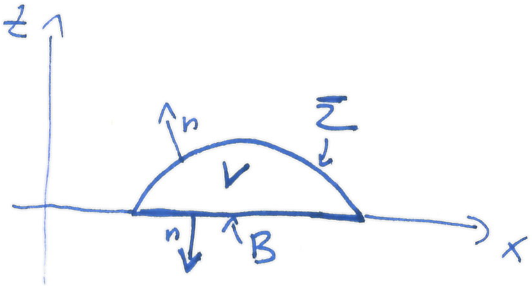
$c = (7, 6, -4)$

$r = 5$

$\Rightarrow$ Trovo il raggio del disco a $z=0$:







Teorema della divergenza:

$$\underbrace{\iiint_V \operatorname{div} F \, dx\,dy\,dz}_{= 0} = \iint_\Sigma F\cdot n \, dS + \iint_B F \cdot n \, dS$$

$$\iint_B F\cdot n\, dS = \int_0^{2\pi}\int_0^3 (0+4)\cdot(-1)\,\rho\, d\rho\, d\vartheta = -4\cdot 9\pi$$

$\Rightarrow$ Flusso di F attraverso $\Sigma = 36\pi$

7 Moltiplicatori di Lagrange:

$$\begin{cases} 1 = \lambda 4x \\ -2 = \lambda(2y+1) \\ 2x^2+y^2+y-2=0 \end{cases} \Rightarrow \frac{1}{4x} = \lambda = \frac{-2}{2y+1}$$

$$\Updownarrow$$

$$2x^2+\left(y+\frac{1}{2}\right)^2 = \frac{9}{4}$$

$$\Rightarrow -8x = 2y+1$$

$$\Rightarrow y = -4x-\frac{1}{2} \quad (*)$$

$$2x^2+\left(-4x-\frac{1}{2}+\frac{1}{2}\right)^2 = \frac{9}{4}$$

$$\Rightarrow 2x^2+16x^2 = \frac{9}{4} \qquad \Rightarrow x = \frac{\pm 1}{2\sqrt{2}}$$

$$(*) \Rightarrow y = \mp\frac{2}{\sqrt{2}}-\frac{1}{2} = \mp\sqrt{2}-\frac{1}{2}$$

P.ti di estremo:

$$A = \left(\frac{1}{2\sqrt{2}}, -\sqrt{2}-\frac{1}{2}\right), \quad B = \left(\frac{-1}{2\sqrt{2}}, \sqrt{2}-\frac{1}{2}\right)$$

massimo: $f(A) = \frac{1}{2\sqrt{2}} + 2\sqrt{2}+1$

minimo: $f(B) = -\frac{1}{2\sqrt{2}} - 2\sqrt{2}+1$

Parte B

(8)

- f differenziabile $\Rightarrow$ f derivabile

  " $\Rightarrow$ f continua

  " $\not\Rightarrow$ $\partial_{x_i} f$ continue

f derivabile $\not\Rightarrow$ f continua

" $\not\Rightarrow$ f differenziabile

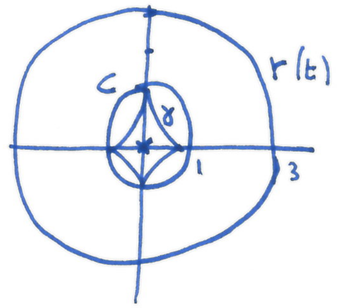
$\partial_{x_i} f$ continue in $U(x)$ $\Rightarrow$ f differenziabile in x.

(9) F è irrotazionale, ma non è definito in $(0,0)$

$\Rightarrow$ $\gamma(t)$ è equivalente a $r(t)$, equivalente a $c(t) = (\cos t, \sin t)$, $t \in [0, 2\pi]$

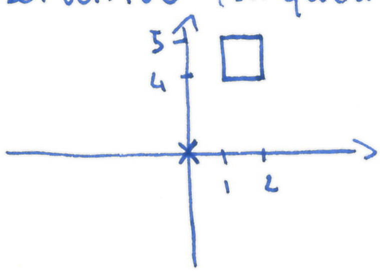
$$\Rightarrow (a) = (b) = \int_0^{2\pi} F(\cos t, \sin t) \cdot (-\sin t, \cos t)\, dt =$$

$$= \int_0^{2\pi} -\sin^2 t - \cos^2 t \; dt = -2\pi.$$



(c): $[1,2] \times [4,5]$ è semplicemente connesso

$\Rightarrow$ F è conservativo (su questo insieme) $\Rightarrow$ $(b) = 0$.



(11) In coordinate polari ho

$$\left| \frac{|\rho \sin\vartheta|^{\alpha} \cos(\rho\cos\vartheta)}{\rho} \right| \leq \rho^{\alpha-1}$$

$\Rightarrow$ se $\alpha > 1 \Rightarrow f$ è continua

se $\alpha \leq 1$ $f(0,y) = |y|^{\alpha-1} \not\to 0$ per $y \to 0$.

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(12) A: 

$$\mathring{A} = \left\{ x^2+y^2 \in (0,1) \cup (1,2) \right\}$$

$$\partial A = \left\{ x^2+y^2 = 0 \right\} \cup \left\{ x^2+y^2 = 1 \right\} \cup \left\{ x^2+y^2 = 2 \right\}$$

$$\bar{A} = \left\{ x^2+y^2 \leq 2 \right\}$$

(13) $\partial_t f(s, g(t), t) = \partial_y f(s, g(t), t) \cdot g'(t) + \partial_z f(s, g(t), t)$

(Indicando $\nabla f = (\partial_x f, \partial_y f, \partial_z f)$)