

$$② E(x, y) = \frac{1}{2} \sum_{j=1}^3 (a_j x + y - b_j)^2$$

• Cerco i punti stazionari, ovvero  $(\bar{x}, \bar{y})$  tale che

$$\nabla E(\bar{x}, \bar{y}) = (0, 0)$$

$$\partial_x E(x, y) = \sum_{j=1}^3 a_j (a_j x + y - b_j)$$

$$\partial_y E(x, y) = \sum_{j=1}^3 (a_j x + y - b_j)$$

Sostituisco  $(a_j, b_j)$  con i valori  $(0, 0)$ ,  $(1, 0)$ ,  $(2, 9)$

$$\Rightarrow \partial_x E(x, y) = (x + y) + 2(2x + y - 9) = 5x + 3y - 18$$

$$\partial_y E(x, y) = y + (x + y) + (2x + y - 9) = 3x + 3y - 9$$

Impongo  $\nabla E(x, y) = (0, 0)$ ,

$$\begin{cases} 5x + 3y - 18 = 0 \\ 3x + 3y - 9 = 0 \end{cases} \Rightarrow \begin{cases} 2x = 9 \\ 3x + 3y = 9 \end{cases} \Rightarrow \begin{cases} x = \frac{9}{2} \\ y = 3 - \frac{9}{2} = -\frac{3}{2} \end{cases}$$

$$\Rightarrow \text{L'unico punto stazionario è } P = \left( \frac{9}{2}, -\frac{3}{2} \right)$$

• Per classificarlo, studio la matrice Hessiana

$$HE(x, y) = \begin{pmatrix} 5 & 3 \\ 3 & 3 \end{pmatrix}$$

$$\det(HE) = 15 - 9 = 6 > 0 \quad \text{tr}(HE) = 5 + 3 = 8 > 0$$

$\Rightarrow P$  è punto di minimo (stretto, relativo).

Inoltre,  $HE$  è def + in  $\mathbb{R}^2 \Rightarrow E$  è convessa

$\Rightarrow P$  è punto di minimo assoluto.

$$3) f(x, y, z) = -2 \left( x^2 + y^2 + z^2 \right)^{-\frac{1}{2}} = \frac{-2}{\sqrt{x^2 + y^2 + z^2}^{\frac{1}{2}}}$$

$$\partial_x f = -2 \cdot \left(-\frac{1}{2}\right) \cdot \left(x^2 + y^2 + z^2\right)^{-\frac{3}{2}} \cdot 2x = \frac{2x}{\left(x^2 + y^2 + z^2\right)^{\frac{3}{2}}}$$

$$\partial_y f = \dots \quad \partial_z f = \dots$$

$$\Rightarrow \nabla f(x, y, z) = \frac{2}{\left(x^2 + y^2 + z^2\right)^{\frac{3}{2}}} \cdot (x, y, z)$$

$$\bullet g(x, y, z) = 4y^2 \Rightarrow \nabla g(x, y, z) = (0, 8y, 0)$$

$$a) e^z \cdot \nabla(f+g)(1, 1, 1) = (0, 1, 0) \cdot \left[ \left(1, 1, 1\right) \cdot \frac{2}{\left(1+1+1\right)^{\frac{3}{2}}} + (0, 8, 0) \right]$$

$$= \frac{2}{3\sqrt{3}} + 8$$

$$b) n = \frac{(x, y, z)}{\sqrt{x^2 + y^2 + z^2}}$$

$$\iint_{\Sigma} \nabla f \cdot n \, dS = \iint_{\Sigma} \frac{2}{\left(x^2 + y^2 + z^2\right)^{\frac{3}{2}}} \cdot \frac{(x, y, z)}{\left(x^2 + y^2 + z^2\right)^{\frac{1}{2}}} \, dS$$

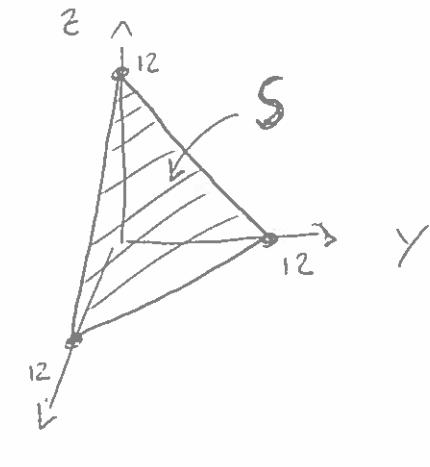
$$= \iint_{\Sigma} \frac{2}{\left(x^2 + y^2 + z^2\right)^2} \, dS = \iint_{\Sigma} \frac{2}{R^2} \, dS = \frac{2}{R^2} \cdot 4\pi R^2 = 8\pi$$

$$d) \operatorname{div}(\nabla g) = 8 \Rightarrow \iiint_V \operatorname{div}(\nabla g) \, dx \, dy \, dz = 8 |V| = 8 \cdot \frac{4}{3}\pi(R^3 - r^3)$$

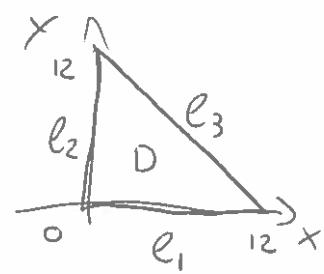
$$= 8 \cdot \frac{4}{3}\pi \cdot 124$$

$$\Rightarrow c) \iint_{\Sigma} \nabla g \cdot n \, dS = \iiint_{\{x^2 + y^2 + z^2 \leq 25\}} \operatorname{div}(\nabla g) \, dx \, dy \, dz = 8 \cdot \frac{4}{3}\pi \cdot 125$$

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(\*)  
CONCLUSIONE:  
minimo: 0  
massimo: 7 · 36



Parametrizzo  $S$ :  $z = 12 - x - y$ , con

$$(x,y) \in D = \{(x,y) \in \mathbb{R}^2 : x+y \leq 12, x \geq 0, y \geq 0\}$$

$$\begin{aligned} \Rightarrow f(x,y,z) &= xy + 2y(12-x-y) + 7x(12-x-y), (x,y) \in D. \\ &= \underbrace{xy - 2xy - 7xy}_{-8xy} - 2y^2 - 7x^2 + 24y + 84x \end{aligned}$$

Cerco i punti stazionari all'interno di  $D$ :

$$\partial_x f = -8y - 14x + 84$$

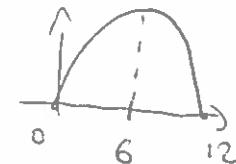
$$\Rightarrow \begin{cases} 7x + 6y = 42 \\ 8x + 6y = 24 \end{cases}$$

$$\partial_y f = -8x - 4y + 24$$

$$\Rightarrow \begin{cases} 7x + 6y = 42 \\ x = -18 \end{cases} \quad \text{Impossibile} \Rightarrow x < 0 \notin D \Rightarrow \begin{array}{l} \text{I punti di massimo e} \\ \text{minimo sono in } \partial D \end{array}$$

Parametrizzo  $\partial D$ :  $l_1: y=0, z=12-x, x \in [0,12]$

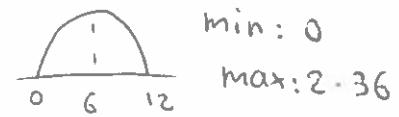
$$F(x,y,z) = 7x(12-x)$$



minimo: 0 in  $(0,0,12)$  e  $(12,0,0)$

massimo: 7 · 36 in  $(6,0,6)$  Analogamente:

$l_2: x=0, z=12-y, y \in [0,12], f = 2y(12-y)$



min: 0

max: 2 · 36

$l_3: z=0, y=12-x, x \in [0,12], F: x(12-x)$  min: 0 max: 36

\*)

⑩ Data la serie  $\sum_{n=1}^{+\infty} f_n(x_1)$ , studia il  $\sup_{x \in [-1,1]} |f_n(x)|$ :

$\boxed{A}$ :  $\sup_{x \in [-1,1]} |f_n(x)| = \frac{1}{n!}$ ,  $\sum \frac{1}{n!}$  converge  $\Rightarrow \boxed{A}$  converge totalmente

$\boxed{B}$ :  $= \frac{1}{2^n}$ .  $\sum \frac{1}{2^n}$  converge  $\Rightarrow \boxed{B}$  //

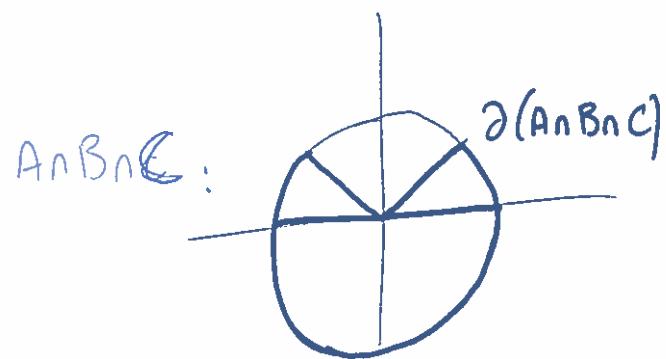
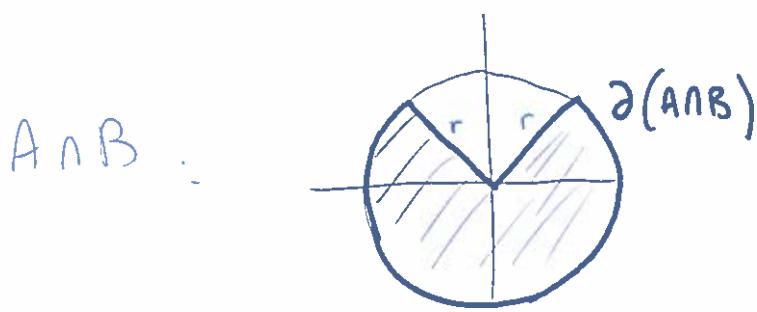
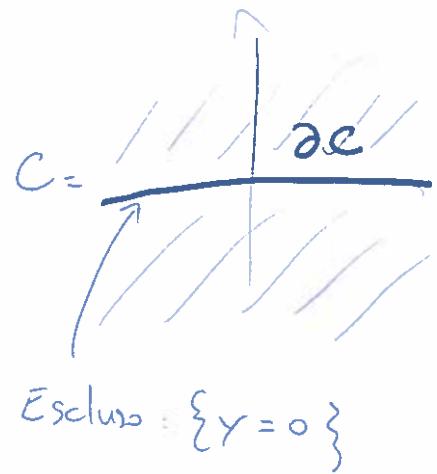
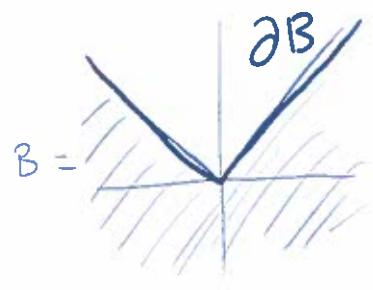
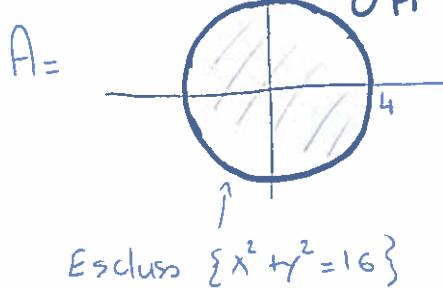
$\boxed{C}$ :  $= \frac{1}{n^{3/2}}$ .  $\sum \frac{1}{n^{3/2}}$  converge  $\Rightarrow \boxed{C}$  //

$\boxed{D}$ :  $= e^{\frac{1}{n}} - 1$   $\sum e^{\frac{1}{n}} - 1 \sim \boxed{\sum \frac{1}{n}}$ , che NON CONVERGE

$\boxed{E}$ :  $= \frac{1}{n} \cdot \sin\left(\frac{1}{n}\right) \sim \frac{1}{n^2} \Rightarrow \boxed{E}$  converge totalmente

$\boxed{F}$ :  $= 1 - \cos\left(\frac{1}{n}\right) \sim \frac{1}{2n^2} \Rightarrow \boxed{F}$

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$$|\partial(A \cap B)| = \frac{3}{4} |\partial A| + r + r$$

$$= \frac{3}{4} \cdot 2r\pi + 2r$$

$$(r=4) = 6\pi + 8$$

$$|\partial(A \cap B \cap C)| =$$

$$= \frac{3}{4} |\partial A| + 2r + 2r$$

$$= 6\pi + 16$$