2. $$x^2+y^2=(x^2+y^2-x)^2$$

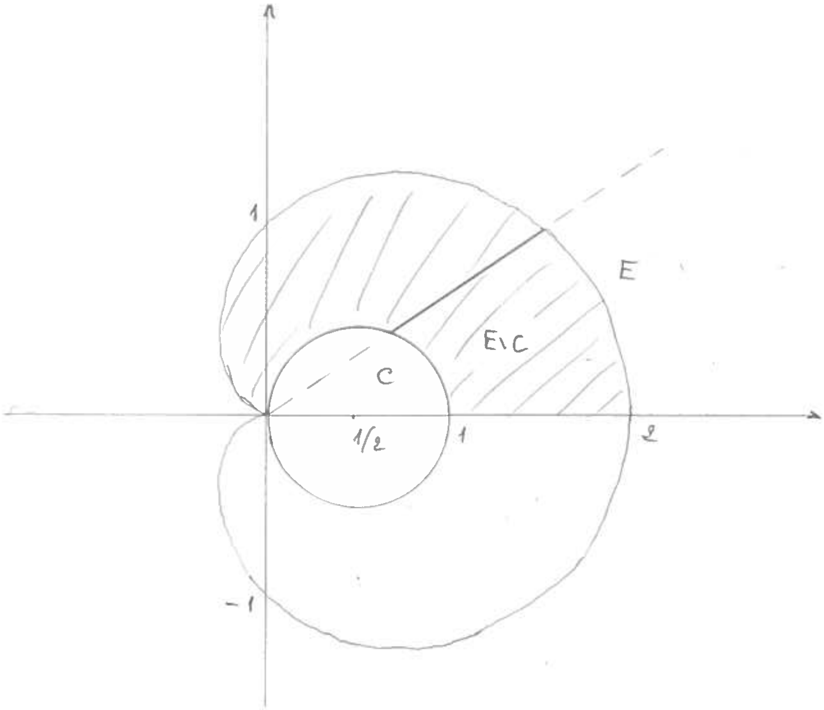
$$\rho^2=(\rho^2-\rho\cos\vartheta)^2$$

$$1=(\rho-\cos\vartheta)^2$$

$$\rho-\cos\vartheta=\pm 1$$

$$\rho=1+\cos\vartheta$$

$$(\rho=-1+\cos\vartheta\le 0)$$



In coordinate cartesiane il cerchio C è dato da

$$\left(x-\tfrac{1}{2}\right)^2+y^2=1/4\ ,$$

$$x^2+y^2-x=0$$

cioè, in coordinate polari,

$$\rho=\cos\vartheta\quad ,\quad -\frac{\pi}{2}\le\vartheta\le\frac{\pi}{2}\ .$$

Allora

$$\iint_{E\setminus C} y\,dx\,dy=\iint_E y\,dx\,dy-\iint_{C\cap\{y\ge 0\}} y\,dx\,dy=$$

$$=\int_0^{\pi}d\vartheta\int_0^{1+\cos\vartheta}\rho^2\sin\vartheta\,d\rho-\int_0^{\pi/2}d\vartheta\int_0^{\cos\vartheta}\rho^2\sin\vartheta\,d\rho$$

$$=\frac{1}{3}\int_0^{\pi}(1+\cos\vartheta)^3\sin\vartheta\,d\vartheta-\frac{1}{3}\int_0^{\pi/2}\cos^3\vartheta\sin\vartheta\,d\vartheta$$

$$=-\frac{1}{12}\Big[(1+\cos\vartheta)^4\Big]_0^{\pi}+\frac{1}{12}\Big[(\cos\vartheta)^4\Big]_0^{\pi/2}=$$

$$=\frac{16}{12}-\frac{1}{12}=\frac{15}{12}=\frac{5}{4}\ .$$

$$\text{area } E=\int_0^{\pi}d\vartheta\int_0^{1+\cos\vartheta}\rho\,d\rho=\frac{1}{2}\int_0^{\pi}(1+2\cos\vartheta+\cos^2\vartheta)\,d\vartheta=$$

$$=\frac{1}{2}\left(\pi+\int_0^{\pi}\frac{1+\cos 2\vartheta}{2}\,d\vartheta\right)=\frac{1}{2}\left(\pi+\frac{1}{2}\pi\right)=\frac{3}{4}\pi$$

e area $(C\cap\{y\ge 0\})=\frac{\pi}{4}\cdot\frac{1}{2}=\frac{1}{8}\pi$

Quindi l'ordinata del baricentro vale: $y_0=\frac{5}{4}\Big/\left(\frac{3}{4}\pi-\frac{\pi}{8}\right)=\frac{5}{4}\Big/\left(\frac{5}{8}\pi\right)=\frac{2}{\pi}\ .$

3. S superficie parametrizzata da

$$\alpha(t,\vartheta) = (t\cos\vartheta,\ t\sin\vartheta,\ \vartheta), \qquad t \in [\sinh 1, \sinh 2]$$
$$\vartheta \in [0, 2\pi]$$

Si tratta di una porzione di elicoide.

$$\alpha_t(t,\vartheta) = (\cos\vartheta, \sin\vartheta, 0), \qquad \alpha_\vartheta(t,\vartheta) = (-t\sin\vartheta, t\cos\vartheta, 1)$$

$$\alpha_t \wedge \alpha_\vartheta = \begin{vmatrix} e_1 & e_2 & e_3 \\ \cos\vartheta & \sin\vartheta & 0 \\ -t\sin\vartheta & t\cos\vartheta & 1 \end{vmatrix} = (\sin\vartheta, -\cos\vartheta, t)$$

Quindi

$$\text{area } S = \int_0^{2\pi} d\vartheta \int_{\sinh 1}^{\sinh 2} \sqrt{\sin^2\vartheta + \cos^2\vartheta + t^2}\, dt =$$

$$= 2\pi \int_{\sinh 1}^{\sinh 2} \sqrt{1+t^2}\, dt$$

Consideriamo il cambiamento di variabile $t = \sinh x$, così che $1+t^2 = 1+\sinh^2 x = \cosh^2 x$.
Risulta

$$\text{area } S = 2\pi \int_1^2 \cosh^2 x\, dx = \int_1^2 \frac{1+\cosh 2x}{2}\, dx = \frac{1}{2}\left[x + \frac{1}{2}\sinh 2x\right]_1^2 =$$

$$= \frac{1}{2}\left(1 + \frac{1}{2}(\sinh 4 - \sinh 1)\right)$$

Sia D la proiezione di S sul piano xy e sia $f: D \to \mathbb{R}$ la funzione il cui grafico è S.
Allora

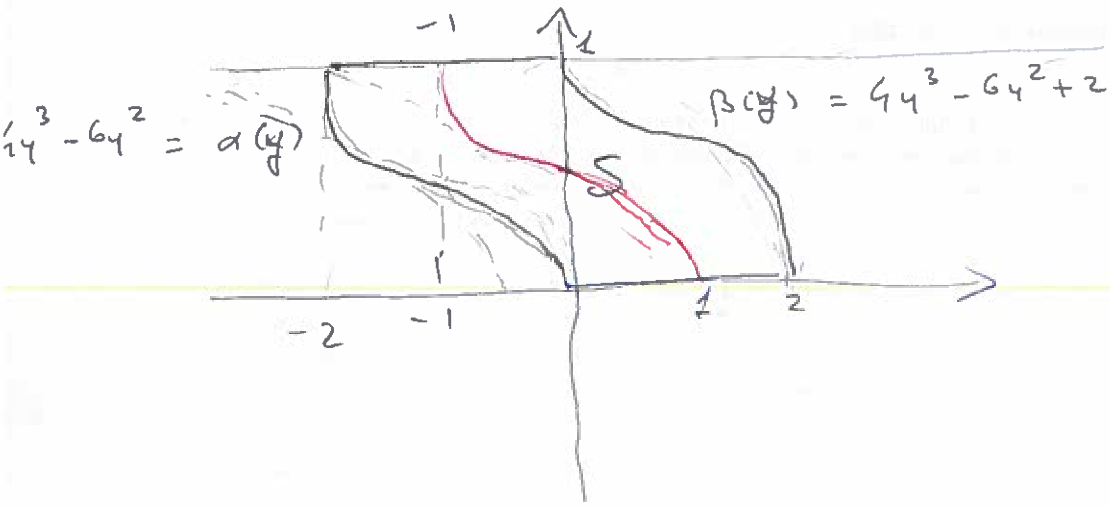
$$\text{vol } V = \iint_D f(x,y)\,dx\,dy = \int_0^{2\pi} d\vartheta \int_{\sinh 1}^{\sinh 2} \vartheta\, \rho\, d\rho =$$

$$= \frac{1}{2}(2\pi)^2 \frac{1}{2}(\sinh^2 2 - \sinh^2 1) = \pi^2 (\sinh^2 2 - \sinh^2 1).$$

$y'(12y(y-1)) = 1$

$(12y^2 - 12y)\, y' = 1$

$4y^3 - 6y^2 = x + c \qquad x = 4y^3 - 6y^2 + k$



$f(x) = k \iff$
$x = -2y^2 + k$

Se $y(0) = 1/2$ ho $\quad 0 = \frac{4}{8} - \frac{6}{4} + k \implies k = 1$

e dunque $x = 4y^3 - 6y^2 + 1 \implies$ $y = 0$ per $x = 1$
$y = 1$ per $x = -1$

$\text{Area}(S) = \int_0^1 dy \int_{\alpha(y)}^{\beta(y)} dx = 2$ come si vede anche elementarmente

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f non ha punti staz. – Ora, $f(x,0) = x$ max in 2 = 2, min in 0 = 0

$f(x,1) = x+2$ max in $0 = 2$, min in $-2 = 0$

$\varphi(y) = f(y, \beta(y)) = 4y^3 - 4y^2 + 2$, mentre $f(y, \alpha(y)) = 4y^3 - 4y^2 = \psi(y)$

$\varphi'(y) = 12y^2 - 8y = 4y(3y-2)$

$\varphi(0) = 2$

$\varphi(2/3) = \frac{32}{27} - \frac{16}{9} + 2 = \frac{38}{27}$

$\varphi(1) = 2$

Ovviamente $\psi(0) = 0$

$\psi(2/3) = -16/27$

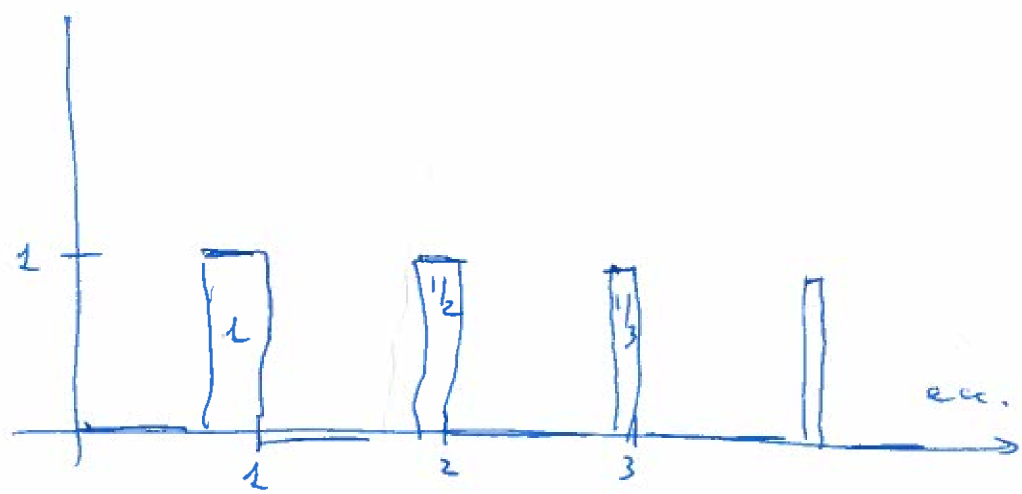
(a) VERO. Infatti la funzione integrale

$F(x) = \int_0^x f(t)\,dt$ è monotona; dunque ammette limite.

(b) FALSO. Controesempio:

$$f(x) = \frac{1}{(x+2)\ln(x+2)}$$

(c) FALSO. Controesempio



(d) FALSO. Un controesempio può essere costruito facilmente a partire dalla serie a segni alterni

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \quad \text{tale che} \quad \sum a_n^2 = +\infty$$

a_n