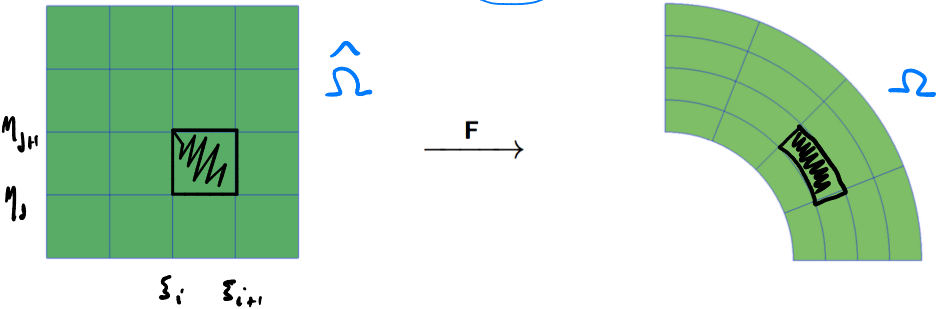
IGA main concepts

The computational domain Ω is parametrized by spline/NURBS:
$\mathbf{F} : \widehat{\Omega} \to \Omega$, where $\mathbf{F}(\xi, \eta) = \sum_i \mathbf{C}_i B_i(\xi, \eta)$, and $\mathbf{C}_i$ are the control points



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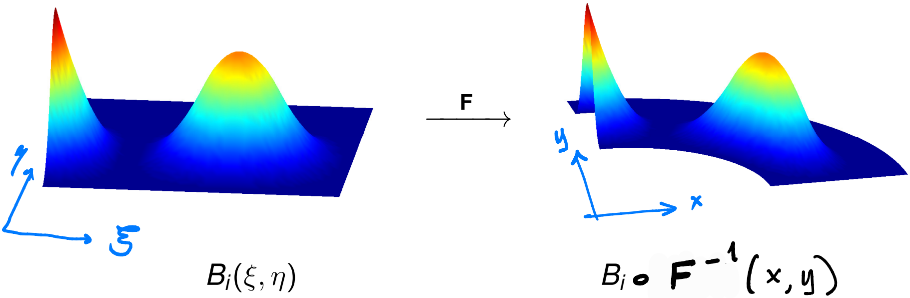

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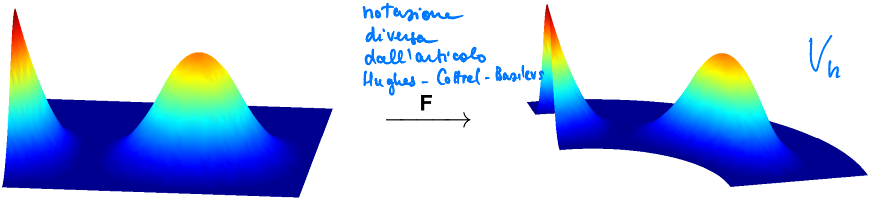
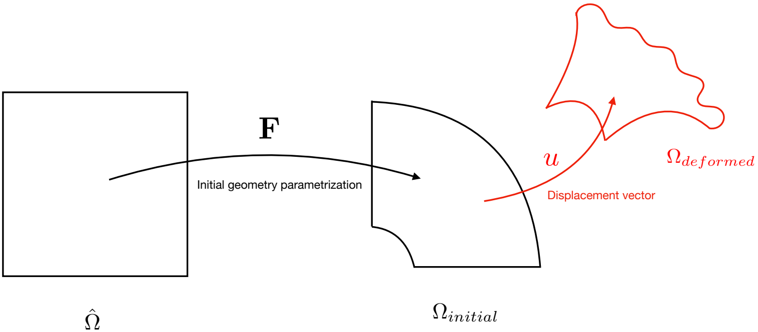


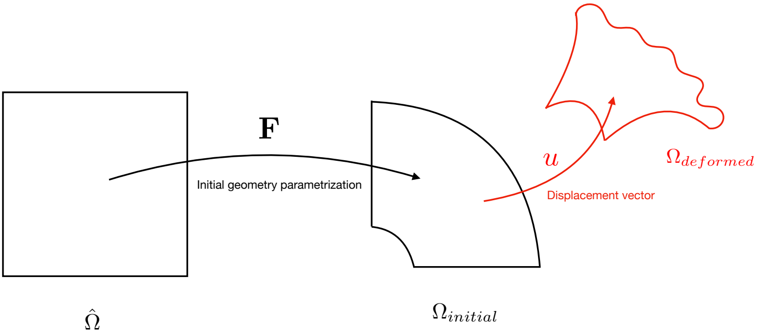
IGA has been proposed by structural engineers:

- **Isoparametric paradigm**: push-forward of splines/NURBS basis functions on the computational domain Ω to approximate the solution of the PDE.
- **Finite Element** kind method (splines etc. replace piecewise polynomials)

Isoparametric concept I



Isoparametric concept I

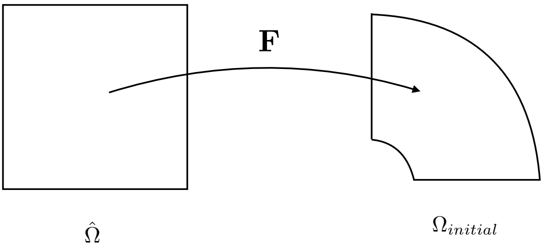


$\Omega_{deformed}$ is parametrized by $(\mathbf{F} + \underbrace{\mathbf{u} \cdot \mathbf{F}}_{\widehat{\mathbf{u}}})$

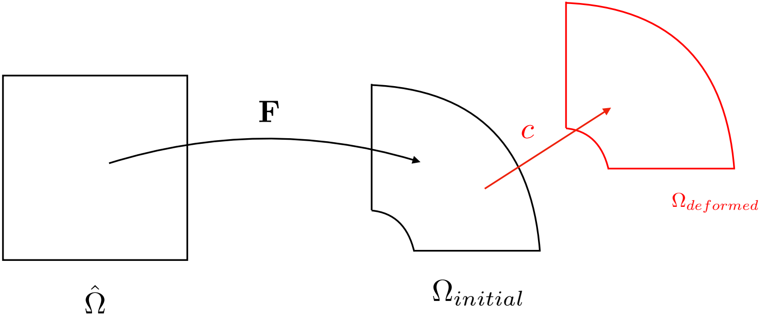
Isoparametric construction

The unknown written w.r.t. the parametric variables ($\widehat{\mathbf{u}} : \widehat{\Omega} \to \Omega_{deformed}$) is represented by the same fuctions that are used for the geometry parametrization $\mathbf{F} : \widehat{\Omega} \to \Omega_{initial}$

Isoparametric concept II

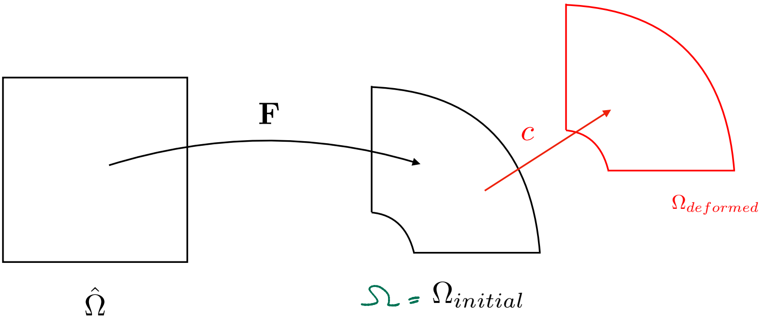


Isoparametric concept II



- a constant $\mathbf{c}$ belongs to the isoparametric space if the same constant $\widehat{\mathbf{c}} = \mathbf{c} \cdot \mathbf{F}$ is represented in the basis over $\widehat{\Omega}$...

Isoparametric concept II



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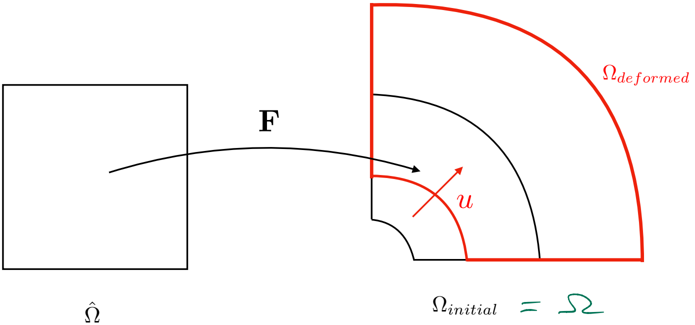
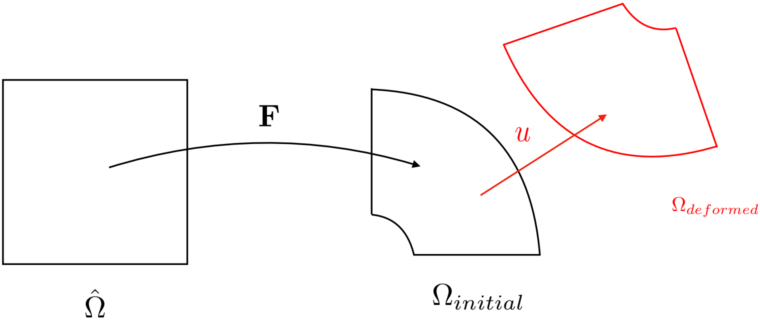
Isoparametric concept II



- a constant **c** belongs to the isoparametric space if the same constant $\widehat{\mathbf{c}} = \mathbf{c} \cdot \mathbf{F}$ is represented in the basis over $\widehat{\Omega}$... ok for splines and NURBS (partition of the unity)
- $\mathbf{u}(x, y) = \mathbf{A} \begin{bmatrix} x \\ y \end{bmatrix}$ is in any isoparametric space since $\widehat{\mathbf{u}}(\xi, \eta) = \mathbf{A}\,\mathbf{F} \begin{bmatrix} \xi \\ \eta \end{bmatrix} = \mathbf{A} \begin{bmatrix} F_1(\xi,\eta) \\ F_2(\xi,\eta) \end{bmatrix}$ is a vector field whose components are linear combination of the parametrization components $F_1(\xi, \eta)$ and $F_2(\xi, \eta)$, and then is represented in the same basis

Isoparametric concept II



The isoparametric space contains any rigid body motion

$$\mathbf{u}(x, y) = \mathbf{c} + \mathbf{R} \begin{bmatrix} x \\ y \end{bmatrix}$$

that form the kernel of the internal elastic energy

IGA main side-effect

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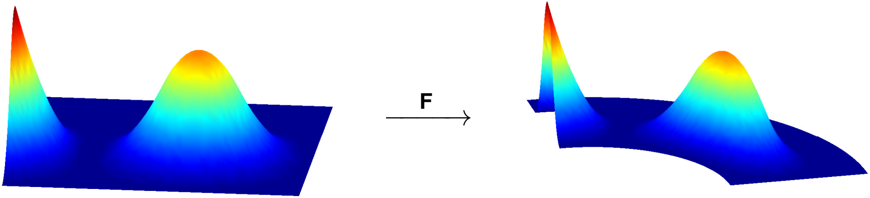

Isogeometric spaces are smooth