Numerical Methods in Engineering Sciences
13/3/2024

First name: _____

Last name: _____

Student ID: _____

☐ I want to take the BASIC EXAM (maximum grade is 24/30)
☐ I want to take the ADVANCED EXAM (maximum grade is 30/30 cum laude)

Total time is 1 hour.

BASIC EXAM

1. Starting from $x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, compute two iterations of the Jacobi method in order to compute an approximate solution of the system $Ax = b$, where

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & -3 \\ 0 & -3 & 6 \end{bmatrix}, \qquad b = \begin{bmatrix} -2 \\ 6 \\ -12 \end{bmatrix}$$

Report intermediate computations.

2. Write the pseudo-code of the Bisection Method. Apply two bisection iterations to find an approximation in $[0, 2]$ of the solution of the equation

$$2x^3 + 4x^2 - x - 6 = 0.$$

ADVANCED EXAM

3. Explain what is the LU factorization of a matrix, without pivoting, and give its pseudo-code. Why pivoting is necessary in the general case? Give an example to support your answer.
Then, compute the LU factorization, without pivoting, of

$$A = \begin{bmatrix} -4 & -4 & -1 \\ -20 & -15 & 0 \\ 0 & 15 & 19 \end{bmatrix}$$

Report the intermediate computations.

4. State and prove the theorem on the existence and uniqueness of the Lagrange interpolation polynomial of a given function.