

- I want to take the BASIC EXAM  
 I want to take the ADVANCED EXAM

Exam rules:

- Basic exam: the maximum grade is 24/30.
- Advanced exam: the maximum grade is 30/30 cum laude.

Total time is 1 hour. Students who get a positive grade in the written part (i.e., at least 18/30) *might* choose to take an oral exam. For students who choose the basic written exam, the maximum grade obtainable can never exceed 24/30.

**BASIC EXAM**

1. With initial guess  $x_0 = 0$  apply one Newton iteration to find an approximate solution of the equation

$$(x^2 + 6x + 4)(x + 2) = 0$$

2. Write the pseudocode of the Gaussian elimination method, without pivoting, and apply it to solve the linear system

$$\begin{bmatrix} 3 & -2 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

showing the intermediate computations.

**ADVANCED EXAM**

3. Write the pseudocode of the bisection method. Apply two bisection iterations to the equation

$$x^2 - 1 = 0 \quad \text{in } [0, 3].$$

4. Introduce the power method for the computation of the dominant eigenvalue and eigenvector, with pseudocode. State and prove its convergence properties. Then, motivate the inverse power method, reporting its pseudocode.