Numerical Methods in Engineering Sciences — Written Exam — First name: _____
3/7/2023 — Last name: _____

Student ID: _____

☐ I want to take the BASIC EXAM
☐ I want to take the ADVANCED EXAM

Exam rules:

- Basic exam: the maximum grade is 24/30.
- Advanced exam: the maximum grade is 30/30 cum laude.

Total time is 1 hour. Students who get a positive grade in the written part (i.e., at least 18/30) *might* choose to take an oral exam. For students who choose the basic written exam, the maximum grade obtainable can never exceed 24/30.

BASIC EXAM

1. Starting from $x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, compute two iterations of the Jacobi method applied to the system $Ax = b$, where

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

Report the intermediate computations.

2. Write the pseudo-code of the composite trapezoidal quadrature rule, then use the composite trapezoidal quadrature rule to compute an approximation of

$$\int_0^{2\pi} \sin^2(t)\, dt$$

by splitting the integration interval $[0, 2\pi]$ into four uniform subintervals. Report the intermediate computations.

ADVANCED EXAM

3. Write the pseudocode of the bisection method. Apply two bisection iterations to the equation

$$x^3 + 3x - 2 = 0 \qquad in\,[0, 1].$$

4. State the convergence theorem of the Newton method for solving nonlinear equations. Prove that the method converge and its order of convergence is 2.