Title: Kinetic theory and the long-time behavior of boundary driven quantum systems near the Zeno limit.

Abstract: We investigate bipartite quantum systems on a Hilbert space $\mathcal{H}_{\mathcal{AB}} = \mathcal{H}_{\mathcal{A}} \otimes \mathcal{H}_{\mathcal{B}}$ where the dynamics is given by two parts: A coherent evolution generated by a Hamiltonain H acting on $\mathcal{H}_{\mathcal{AB}}$, and a Lindbladian dissipator $\mathcal{D} = \mathcal{D}_{\mathcal{A}} \otimes \mathcal{I}_{\mathcal{B}}$ acting non-trivially only on operators on $\mathcal{H}_{\mathcal{A}}$. The evolution equation we study is

$$\dot{\rho}(t) = -i[H, \rho(t)] + \gamma \mathcal{D}\rho(t) \ , \qquad (*)$$

where γ is a constant taken to be large. The limit $\gamma \to \infty$ is known as the Zeno limit. We assume that $\mathcal{D}_A$ is ergodic with a unique steady state π_A and has a spectral gap. It is known that under these conditions, after a short initial layer, solutions $\rho(t)$ of $(*)$ satisfy $\rho(t) = \mu_A \otimes \mu(t) + \mathcal{O}(\gamma^{-1})$ for some density matrix $\mu(t)$ on $\mathcal{H}_B$. Understanding the evolution of $\mu(t)$ is the key to understanding the long time behavior of solutions to $(*)$, and in particular, its steady state solutions.

There is a strong and fruitful analogy with the theory of hydrodynamic limits. One may think of $(*)$ as an analog of the Boltzmann equation with γ^{-1} corresponding to the Knudsen number, which one takes to zero in the hydrodynamic limit. One may think of $\mu(t)$ as corresponding to the hydrodynamic moments. In the kinetic setting, making appropriate rescalings of space and time involving the Knudsen number, one obtains the Euler equations or some version of the Navier-Stokes equations, depending on the rescaling. Note that these hydrodynamic equations do not involve the Knudsen number, which drops out due its appearance in the rescaling so that all terms in the kinetic equation pick up a common power of it.

We apply kinetic methods to derive a “hydrodynamic” description of the evolution described by $(*)$ near the Zeno limit, and use this to study the steady states of $(*)$. In particular, study an expansion for the steady states, finding an equation for the starting point, and we prove that the expansion is convergent for large enough γ. The expansion is a close analog of the Hilbert expansion of kinetic theory. This is joint work with David Huse and Joel Lebowitz.