INdAM workshop

RECENT ADVANCES IN KINETIC EQUATIONS AND APPLICATIONS

Rome, Italy, 11 November - 15 November 2019

BOOK OF ABSTRACTS

TALKS

Anton Arnold
Short- and long-time behavior in (hypo)coercive ODE-systems and Fokker-Planck equations

We are concerned with the short- and large-time behavior of Fokker-Planck equations with linear drift, i.e. $\partial_t f = div(D\nabla_x f + Cxf)$. A coordinate transformation can normalize these equations such that the diffusion and drift matrices are linked as $D = C_s$, the symmetric part of C.

The first main result of this talk is the connection between normalized Fokker-Planck equations and their drift-ODE $\dot{x} = -Cx$: Their L^2-propagator norms actually coincide. This implies that optimal decay estimates on the drift-ODE (w.r.t. both the maximum exponential decay rate and the minimum multiplicative constant) carry over to sharp exponential decay estimates of the Fokker-Planck solution towards the steady state.

Secondly, we define an "index of hypocoercivity", both for ODEs and Fokker-Planck equations that describes the interplay between between the dissipative and conservative part of their generator. This index characterizes the polynomial decay of the propagator norm for short time.

References

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⋆ A. Arnold, C. Schmeiser, B. Signorello. Sharp decay estimates and L^2-propagator norm for Fokker-Planck equations with linear drift, preprint 2019.
⋆ A. Arnold, J. Erb. Sharp entropy decay for hypocoercive and non-symmetric Fokker-Planck equations with linear drift, arXiv 2014.
⋆ F. Achleitner, A. Arnold and D. Stürzer: Large-time behavior in non-symmetric Fokker-Planck equations; Rivista di Matematica della Università di Parma 6 (2015) 1-68.

Luigi Barletti, Philipp Holzinger & Ansgar Jüngel
Derivation of quantum drift-diffusion equations for a spin-orbit 2D electron gas

We derive quantum drift-diffusion equations (QDDE) for a 2-dimensional electron gas with spin-orbit interaction of Rashba type. The (formal) derivation turns out to be a non-standard application of the usual mathematical tools (Wigner transform, Moyal product expansion, Chapman-Enskog expansion). The main peculiarity consists in the fact that a non vanishing current is already carried by the leading-order term in the Chapman-Enskog expansion. Even though the method is general, in order to obtain explicit expressions we work in a regime of small spin polarization. To our knowledge, this is the first example of QDDE involving the full (four components) spin structure. Indeed, previous models were either quantum bipolar (two components) or full-spin semiclassical (leading-order in the Moyal expansion).

Marzia Bisi & Romina Travaglini
A BGK model for mixtures of monoatomic and polyatomic gases

The generalization of the classical BGK model to a gas mixture is not obvious (and not unique), since there appear several free parameters in the Maxwellian attractors that could be properly chosen in order to reproduce the basic features of the Boltzmann equations that one wants to approximate, namely the Maxwellian equilibria, the validity of correct conservation laws, and the H-theorem. The BGK models available in the literature may be divided into two classes. A first way of modelling, which dates back to Andries, Aoki and Perthame (2002), assumes that in each kinetic equation a single relaxation-type operator describes the effects of collisions with all the other species. A second option assumes collision operators made up by a sum of binary BGK terms, preserving thus the structure of the original Boltzmann system.
In this talk, we present a recent BGK model valid for mixtures of monoatomic and polyatomic species. Each polyatomic constituent is characterized by its proper number of internal energy levels (to mimic non-translational degrees of freedom), generalizing thus other kinetic models which assume the same fixed number of energy levels for each polyatomic gas. We propose a system of BGK equations showing

a unique relaxation operator for each component, but many technicalities appear in the proofs of the consistency of the model, since each particle must be identified by a pair of indices, denoting the gas which it belongs to and its energy level.

Laurent Boudin
Some mathematical properties of the Boltzmann equation for mixtures

Emeric Bouin
TBA

José Alfredo Cañizo
The scaling hypothesis for Smoluchowski's coagulation equation with bounded perturbations of the constant kernel

We show that universal scaling behaviour takes place for solutions with finite-mass and finite second moment to Smoluchowski's coagulation equation, when small bounded perturbations of the constant kernel are considered. All constants can be explicitly estimated. This is a work in collaboration with Sebastian Throm.

José Antonio Carrillo
Swarming models with local alignment effects: phase transitions & hydrodynamics

We will discuss a collective behavior model in which individuals try to imitate each others'velocity and have a preferred asymptotic speed. It is a variant of the well-known Cucker-Smale model in which the alignment term is localized. We showed that a phase change phenomenon takes place as diffusion decreases, bringing the system from a 'disordered" to an "ordered" state. This effect is related to recently noticed phenomena for the diffusive Vicsek model. We analysed the expansion of the large friction limit around the limiting Vicsek model on the sphere leading to the so-called Self-Organized Hydrodynamics (SOH). This talk is based on papers in collaboration with Aceves-Sanchez, Barbaro, Bostan, Cañizo and Degond.

Guido Cavallaro
The Vlasov-Poisson equation with infinite mass

Some recent results on the initial value problem for the Vlasov-Poisson equation (which models a plasma) in an unbounded domain are presented. The key point is that the mass (the integral of the density function) is allowed to be unbounded, in order to show that the dynamics has essentially a local character and it is not much influenced by the size of the system. Some physical situations are discussed, as the evolution in presence of external fields and the evolution for a plasma constituted by two species with opposite charge sign.

Frédérique Charles
Mathematical and Numerical study of a dusty knudsen gas mixture

We consider a mixture composed of a gas and dust particles in a very rarefied setting. Whereas the dust particles are individually described, the surrounding gas is treated as a Knudsen gas, in such a way that interactions occur only between gas particles and dust by means of diffuse reflection phenomena. I will present the model, the existence and the uniqueness of the solution to this model, and provide a numerical strategy for the study of the equations. At the numerical level, we focus our attention on the phenomenon of energy transfer between the gas and the moving dust particles. This is a common work with Francesco Salvarani.

Laurent Desvillettes

H-theorem for some extensions of the Boltzmann operator

We use the strategy devised for proving a dissipation entropy estimate for the Landau equation (with Coulomb interaction) in order to obtain new H-theorems for complex collision kernels. We present in particular the issues of the kernels corresponding to "$2 \leftrightarrow 1 + 1$" collisions in operators related to weak turbulence.

Jean Dolbeault

L^2 hypocoercivity, inequalities and applications

The purpose of the L^2 hypocoercivity method is to obtain rates for solutions of linear kinetic equations without regularizing effects, in asymptotic regimes. Initially intended for systems with compactness or confinement in position space and simple local equilibria, the method has been extended to various local equilibria in velocities and non-compact situations in positions. It is also flexible enough to include non-local transport terms associated with Poisson coupling. The lecture will be devoted to a review of some recent results which rely on various, deep functional inequalities. An application to the linearized Vlasov-Poisson-Fokker-Planck system will also be presented.

Klemens Fellner

Oscillatory solutions for nonlinear Becker-Döring type models

In view of recent static light scattering measurements of depolymerisation experiments in prion dynamics, we present and analyse bi-monomeric nonlinear Becker-Döring type models, which feature sustained oscillation.

Amic Frouvelle

Body-attitude alignment: link with rodlike polymers, quaternions and phase transition

We present a model of alignment of individuals based on body attitude (birds aligning their heading and wings directions for instance). The kinetic model in consideration is a BGK-type model for which the velocity variable is a rotation matrix of dimension 3. We present an interesting link between this model and a generalization of the Maier-Saupe model for alignment of diluted rodlike polymers in dimension 4, due to the fact that a rotation can be represented by a unit quaternion (or its opposite, which relates to the fact that a rodlike polymer is unoriented). We obtain the phase diagram of this model: when the alignment strength is low, the uniform distribution is the only equilibria, when the strength is sufficiently large, there exists a unique family of stable (concentrated) distributions, and in between, we have stability of both non-aligned and aligned states. This comes from works in collaboration with Pierre Degond, Antoine Diez, Sara Merino-Aceituno and Ariane Trescases.

Irene Gamba

Collisional Kinetics of Multi-Component System Models

We study the mathematical properties of complex particle systems modeling the 'binary mixing of gas mixtures'. More precisely, we focus on the interaction of monoatomic and polyatomic gases with different masses. The model is realized by a Boltzmann system of equations for the evolution of vector valued probability distribution densities describing the random interacting particles through non-local bilinear forms, corresponding to the dynamics of binary mixing of identical shape particles with internal energy exchange but different masses. The corresponding Cauchy problem takes place in Banach spaces naturally associated to the solution observables, yielding global existence and uniqueness of vector valued solutions systems in L^p_K-spaces, for $1 \leq p \leq \infty$ with clearly distinguished initial data depending on their diverse mass parameters. We also discuss numerical approximating conservative schemes that can be shown to converge thanks to the regularity properties associated to the underlying system. This is work in collaboration of Milana Pavic-Colic and Erica de la Canal.

François Golse
TBA

Maria Groppi
Consistent BGK models for gas mixtures and their hydrodynamic limits

Kinetic BGK models are often used in various applications in rarefied gas dynamics and plasma physics, because of the complexity of nonlinear Boltzmann-type kinetic equations for the description of gaseous systems involving particles od different species. In this talk, some consistent relaxation time-approximation models of BGK-type for inert gas mixtures are presented and their main properties are discussed [$\star$,$\star\star$]. Consistency means three basic properties: correct reproduction of conservation laws, H-theorem and uniqueness of equilibrium solution.
The main peculiarities of the presented BGK models will be highlighted with reference to their continuum limits obtained by Chapman-Enskog expansions [$\star\star\star$]. In particular, it will be shown that a recent BGK model [$\star\star$], reproducing the structure of the Boltzmann collision operator for mixtures and well suited to deal with various intermolecular collisional potentials, can lead in the hydrodynamic limit (in a proper collision dominated regime) to multitemperature and multivelocity Euler and Navier Stokes equations.
(Joint work with M. Bisi, G. Martalò, and G. Spiga, Department of Mathematical, Physical and Computer Sciences, University of Parma).

References

$\star$ M. Groppi, G. Russo, G. Stracquadanio, Semi-Lagrangian approximation of BGK models for inert and reactive gas mixtures, in "From Particle Systems to Partial Differential Equations V", Springer Proceedings in Mathematics and Statistics 258, Patricia Gonçalves and Ana Jacinta Soares (Eds.) (2018), p. 53-80.
$\star\star$ A.V. Bobylev, M. Bisi, M. Groppi, G. Spiga, I.F. Potapenko, A general consistent BGK model for gas mixtures, *Kinet. Relat. Models* **11** (2018), 1377–1393.
$\star\star\star$ M. Bisi, A.V. Bobylev, M. Groppi, G. Spiga, Hydrodynamic Equations from a BGK Model for Inert Gas Mixtures, AIP Conference Proceedings,AIP Conf. Proc. 2132, 130010-1130010-8 (2019).

Daniel Han-Kwan
Asymptotic stability of homogeneous equilibria for screened Vlasov-Poisson systems

We shall describe a recent alternative proof of the Landau Damping result on the whole space obtained by Bedrossian, Masmoudi and Mouhot. Our approach is based on the method of characteristics and on the derivation of pointwise in time dispersive estimates for the linearized equation. Joint work with T. Nguyen (Penn State) and F. Rousset (Orsay).

Mikaela Iacobelli
Well-posedness and singular limits for the VPME system

The Vlasov-Poisson system with massless electrons (VPME) is widely used in plasma physics to model the evolution of ions in a plasma. It differs from the classical Vlasov-Poisson system (VP) in that the Poisson coupling has an exponential nonlinearity that creates several mathematical difficulties. We will discuss a recent result proving uniqueness for VPME in the class of solutions with bounded density, and global existence of solutions with bounded density for a general class of initial data, generalising to this setting all the previous results known for VP. Moreover we will talk about a mean field derivation of the VPME and a rigorous quasi neutral limit for initial data that are close to analytic data deriving the Kinetic Isothermal Euler (KIE) system from the VPME in dimensions d=1,2,3. Lastly, we combine these two singular limits in order to show how to obtain the KIE system from an underlying particle system.

Juhi Jang
The Kinetic Fokker-Planck Equation in Bounded Domains

I will discuss the kinetic Fokker-Planck equation with inelastic boundary conditions in one-space dimension with focus on the structure of solutions near the singular set. The inelastic boundary conditions are characterized by a restitution coefficient r describing the amount of energy lost in the collisions of the particles with the boundaries of the domain. I will present the non-uniqueness result for r less than the critical value and the uniqueness for r greater than the critical value. The talk is based on a joint work with H.J. Hwang and J.J.L. Velázquez.

Ansgar Jüngel

Maxwell-Stefan models for fluid mixtures: derivation, analysis, stochastics

The Maxwell-Stefan equations describe the dynamics of fluid mixtures in a diffusive regime. Examples include heliox for asthma, ion transport in biological membranes, and dynamics of lithium-ion batteries. Maxwell-Stefan systems consist of cross-diffusion equations with generally nonsymmetric, indefinite diffusion matrices. Written in terms of chemical potentials or entropy variables, the diffusion matrix of the transformed system (the so-called Onsager matrix) becomes symmetric and positive semi-definite, which is the starting point for the analysis.
In this talk, we present some approaches to derive the Maxwell-Stefan equations rigorously from Boltzmann systems in the diffusion limit or from Euler flow systems in the high-friction limit. We recall some results on the existence of global weak solutions, using the boundedness-by-entropy method, and extend this method to Maxwell-Stefan systems with stochastic forcing.

Chanwoo Kim

On regularity of Boltzmann equation in bounded domains.

We will discuss on recent results of regularity theory of Boltzmann equation in bounded domains in various circumstances.

Matthieu Léautaud

Approximate controllability of hypoelliptic equations

In this talk we consider controllability issues for hyperbolic and parabolic equations involving a hypoelliptic operator (a sum of squares of vectorfields). We also discuss the minimal mass left by eigenfunctions of such an operator on subdomains, in the high-frequency limit. This is a joint work with Camille Laurent.

Bertrand Lods

Long time dynamics for the Landau-Fermi-Dirac equation with hard potentials

In this joint wok with Ricardo Alonso (PUC-Rio, Brazil) and Véronique Bagland (Université Clermont-Auvergence, France) we discuss the long time behaviour for the homogeneous Landau-Fermi-Dirac equation in the hard potential case. Uniform in time estimates for statistical moments and Sobolev regularity are presented and used to prove exponential relaxation of non degenerate distributions to the Fermi-Dirac statistics. All these results are valid for rather general initial datum. An important feature of the estimates is the independence with respect to the quantum parameter. Consequently, in the classical limit the same estimates are recovered for the Landau equation.

Stéphane Mischler

On subgeometric Doeblin-Harris Theorem

Olga Mula Hernández

Solving the neutron transport equation with certified error control

In the numerical solution of kinetic problems, one often tacitly assumes that the computed output represents the corresponding continuous solution reasonably well, without being, however, able to actually quantify its quality in any rigorous sense. Often interest shifts then towards accurately solving the (fixed) discrete problem which by itself may pose significant challenges. Instead, we present a recent contribution where the central objective is to put forward a new algorithmic paradigm where the deviation of the numerical result from the exact continuous solution is certifiably quantified with respect to a problem relevant norm. The strategy has been developed in the framework of the neutron transport equation but we will outline its generality and potential to address other types of kinetic problems. In addition to this, we will discuss another challenging point for numerical computations which stems from the oscillatory behavior of cross sections with respect to the energy of the particles: we present a homogenization result for the neutron transport equation which shows that the limiting equation has an involved memory term which poses important difficulties for numerical computations.

Stefano Olla
Kinetic and hydrodynamic limits for a chain of harmonic oscillators with a point Langevin heat bath

We consider an infinite chain of coupled harmonic oscillators with a Langevin thermostat attached at the origin and energy, momentum and volume conserving noise that models the collisions between atoms. The noise is rarefied in the limit so that in the macroscopic unit time only a finite number of collisions takes place. We prove that after the hyperbolic space-time rescaling the Wigner distribution, describing the energy density of phonons in space-frequency domain, converges to a positive energy density function $W(t, y, k)$ that evolves according to the linear kinetic equation, with the interface condition at $y = 0$ that corresponds to reflection, transmission and absorption of phonons.

Under a further superdiffusive rescaling we prove that the macroscopic evolution of the thermal energy is givrened by a fractional heat equation with a special boundary condition at the position of the heat bath.

The results have been obtained in collaboration with T. Komorowski (Polish Academy of Sciences), L. Ryzhik (Stanford Univ.) and H. Spohn (TU München).

Nastassia Pouradier-Duteil
Control to achieve or prevent consensus of opinion-formation models

We first elaborate control strategies to prevent clustering effects in opinion formation models. This is the exact opposite of numerous situations encountered in the literature where, on the contrary, one seeks controls promoting consensus. In order to promote declustering, instead of using the classical variance that does not capture well the phenomenon of dispersion, we introduce an entropy-type functional that is adapted to measuring pairwise distances between agents. We then focus on a Hegselmann-Krause type system and design declustering sparse controls both in finite-dimensional and kinetic models. We provide general conditions that characterize whether clustering can be avoided as function of the initial data. Such results include the description of black holes (where complete collapse to consensus is not avoidable), safety zones (where the control can keep the system far from clustering), basins of attraction (attractive zones around the clustering set) and collapse prevention (when convergence to the clustering set can be avoided). In a second part, we present a modified version of the Hegselmann-Krause opinion dynamics, in which the weights of agents are allowed to evolve in time. We elaborate control strategies to achieve consensus at a given target by acting on the weights of agents.

Christian Schmeiser
A kinetic model for myxobacteria alignment and reversal interactions

Experiments with colonies of myxobacteria are modeled as ensembles of rod shaped particles moving with constant speed in two dimensions, interrupted by hard binary collisions leading to alignment or velocity reversal. Results on nematic alignment in the spatially homogeneous case will be presented. For a regularized problem existence can be shown for the dynamic problem and for equilibria.

Ana Jacinta Soares
Kinetic modelling of autoimmune diseases

We construct a mathematical model of kinetic type in order to describe the microscopic interactions between cells that are involved in autoimmune diseases. Three cell populations are considered and the distribution function of each population depends on the biological activity variable defining the functional state relevant for that population. We study the wellposedness of our kinetic system and complement our study with some numerical tests illustrating the sensitivity of the model to certain parameters with biological significance.

Work in collaboration with M. P. Ramos and C. Ribeiro.

Eric Sonnendrücker
Gyrokinetic modeling and simulation

In a strong and slowly varying magnetic field, the Vlasov-Maxwell equations can be approximated by the so-called gyrokinetic model, that has only 5 phase spaces variables, one of them being a physical invariant. This model is now the workhorse of Tokamak simulations used for magnetic confinement fusion. In this talk we will present a derivation of this model by asymptotic analysis of the characteristics for a given electromagnetic field and then derive a variational principle for the nonlinear model coupling the distribution function and the electromagnetic fields. This variational principle is then directly discretised in phase space with tools from Finite Element Exterior Calculus and the Particle In Cell method. This yields a large set of coupled ordinary differential equations that has a Poisson structure and exact invariants that can be approximately preserved for long times using geometric numerical integration techniques for ordinary differential equations.

Satoshi Taguchi
Motion of a slightly rarefied gas induced by a discontinuous wall temperature

In this talk, we discuss a motion of a rarefied gas in a long and straight two-dimensional channel caused by a discontinuous wall temperature. More precisely, the channel is subdivided into two infinitely long subchannels and the walls of each subchannel are kept at an equal and uniform temperature so that the wall temperature of the whole channel is discontinuous at the point of the subdivision. Under the assumption that the jump in the wall temperature is small, the steady behavior of the gas induced by the discontinuous wall temperature is investigated on the basis of the linearized Boltzmann equation and the diffuse reflection boundary condition in the case where the Knudsen number, defined by the ratio of the molecular mean free path and the width of the channel, is small. Using a matched asymptotic expansion method combined with Sone's asymptotics for small Knudsen numbers, a Stokes system describing the overall macroscopic behavior of the gas inside the channel is derived, with a new feature of a "slip boundary condition" for a flow due to the jump discontinuity in the boundary temperature. It takes the form of a diverging singularity with source and sink located at the points of discontinuity, whose strength is determined through the analysis of a spatially two-dimensional Knudsen layer (or Knudsen zone) problem.

Shigeru Takata
A revisit to the Cercignani–Lampis model: Langevin picture and its numerical simulation

The Cercignani–Lampis (CL) model for the gas–surface interaction is revisited from the Langevin dynamics viewpoint. By a straightforward time-dependent extension of Cercignani's Fokker–Planck formalism, the corresponding Langevin equation is identified. The Langevin description sheds light on the dynamical feature of the stochastic process corresponding to the CL model. Numerical simulation is performed as well to reproduce the scattering kernel and reflection intensity distribution numerically. Due to the property of the noise in the stochastic process, the Euler–Maruyama scheme does not work well from a practical viewpoint. The Milstein scheme is used to achieve a satisfactory numerical convergence with respect to the time discretisation. This is a joint work with S. Akasobe and M. Hattori.

Daniela Tonon
On the inhomogeneous Boltzmann equation for hard potential without cut-off

In this talk, we investigate the problems of Cauchy theory and exponential stability for the inhomogeneous Boltzmann equation without angular cut-off, in a physically relevant framework that include the case of hard potentials with moderate angular singularity. We show a result of existence and uniqueness of solutions in a close-to-equilibrium regime for this equation in weighted Sobolev spaces with a polynomial weight, whereas all the previous works on the subject, were developed with a weight prescribed by the equilibrium. Moreover, we show an exponential stability for such a solution, with a rate as close as we want to the optimal rate given by the semigroup decay of the linearized equation.

POSTERS

Giorgio Martalò
The Evaporation-Condensation Problem for a Binary Mixture of Rarefied Gases

Kinetic theory is the classical framework to describe the dynamics of rarefied gases. However, some particular problems, like the half space problem of evaporation and condensation, can be initially investigated at the macroscopic level by means of hydrodynamic equations; such analysis can provide also some qualitative indications about kinetic solutions, even when the two approaches give solutions that do not coincide accurately (out of the boundary layer). By using typical qualitative methods of dynamical systems theory, we will discuss from a mathematical point of view the main features of the evaporation-condensation problem for a binary mixture of rarefied gases modeled by a set of Navier-Stokes equations, obtained as hydrodynamic limit of a recent BGK description. Some numerical results for a mixture of noble gases will be presented and discussed.

Edoardo Zoni
Gyrokinetic theory with polynomial transforms: a model for ions and electrons in maximal ordering

Tobias Wöhrer
Large time behaviour in defective Fokker-Planck equations

Funding sources:
Istituto Nazionale di Alta Matematica "Francesco Severi"
Università degli Studi di Pavia, Dipartimento di Matematica 'Felice Casorati"
Ambassade de France en Italie, Programme Cassini Senior
Agence Nationale de la Recherche